Design and control of a 3 dof model to predict the performance at the OBSPM

Jennifer Watchi

Christophe Collette

1. Design of the control law for centralized and decentralized control
   - Design of the controller
   - Stability check

2. Performance

Reminder: description of the model

- Sensor S1 and S2: measures motion in x and z directions

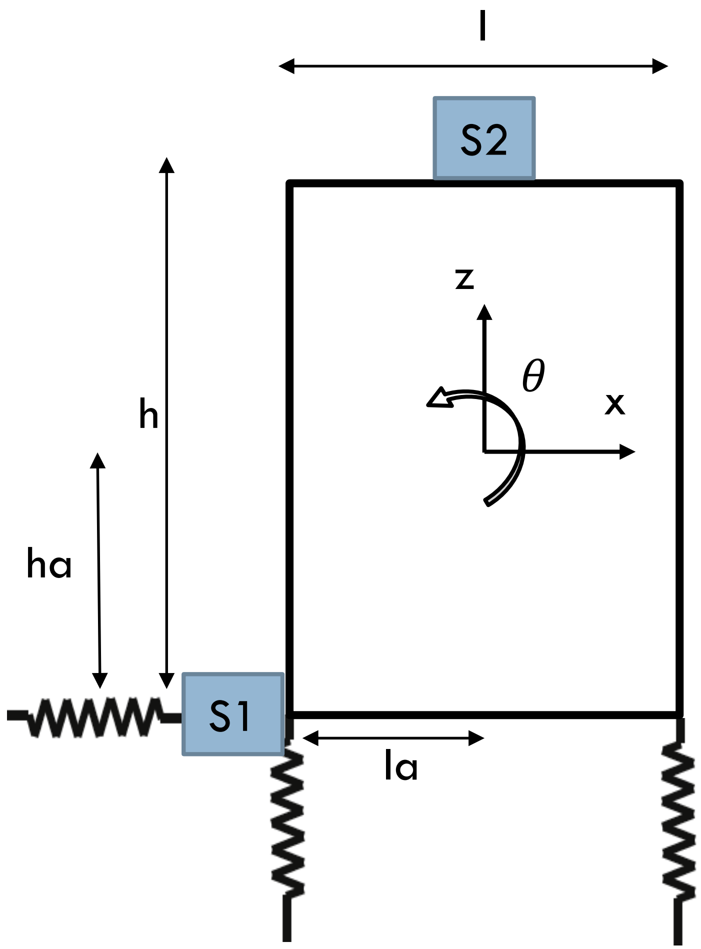
- Actuators collocated with the springs

- Properties

l = 0.8 m = la; h = 1.7 m = ha

m = 400 kg; k = 15e3 N/m;

I = 115 kg m^2

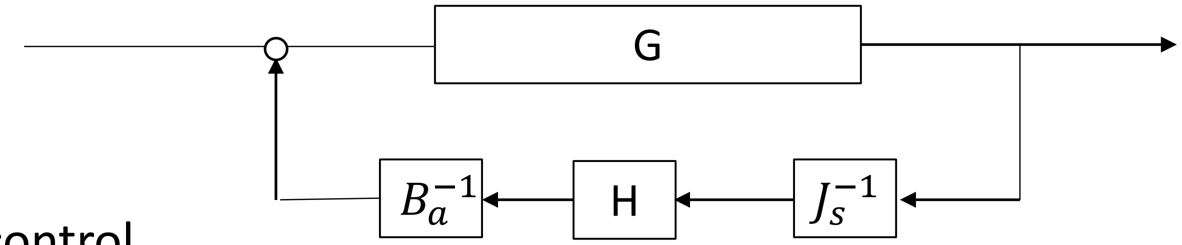


Centralized and decentralized control

We are going to test two different control laws: centralized and decentralized control.

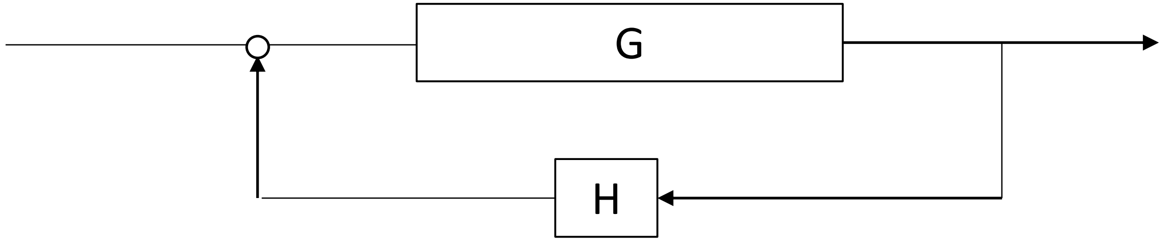
- Centralized control

The signal measured by the sensor (in the local coordinates) is projected into the global coordinates thanks to the jacobian (J_s^{-1}). The controller is applied in this coordinates and the control signal is then reprojected into the coordinates of the actuators (B_a^{-1}).



- Decentralized control

The controller is applied on the signal measured by the sensor in the local coordinates. The control signal is then injected to the actuators.



Design of the controller: plot the plant*gain

- The plant matrix shows the transfer functions between the actuators and the sensors.

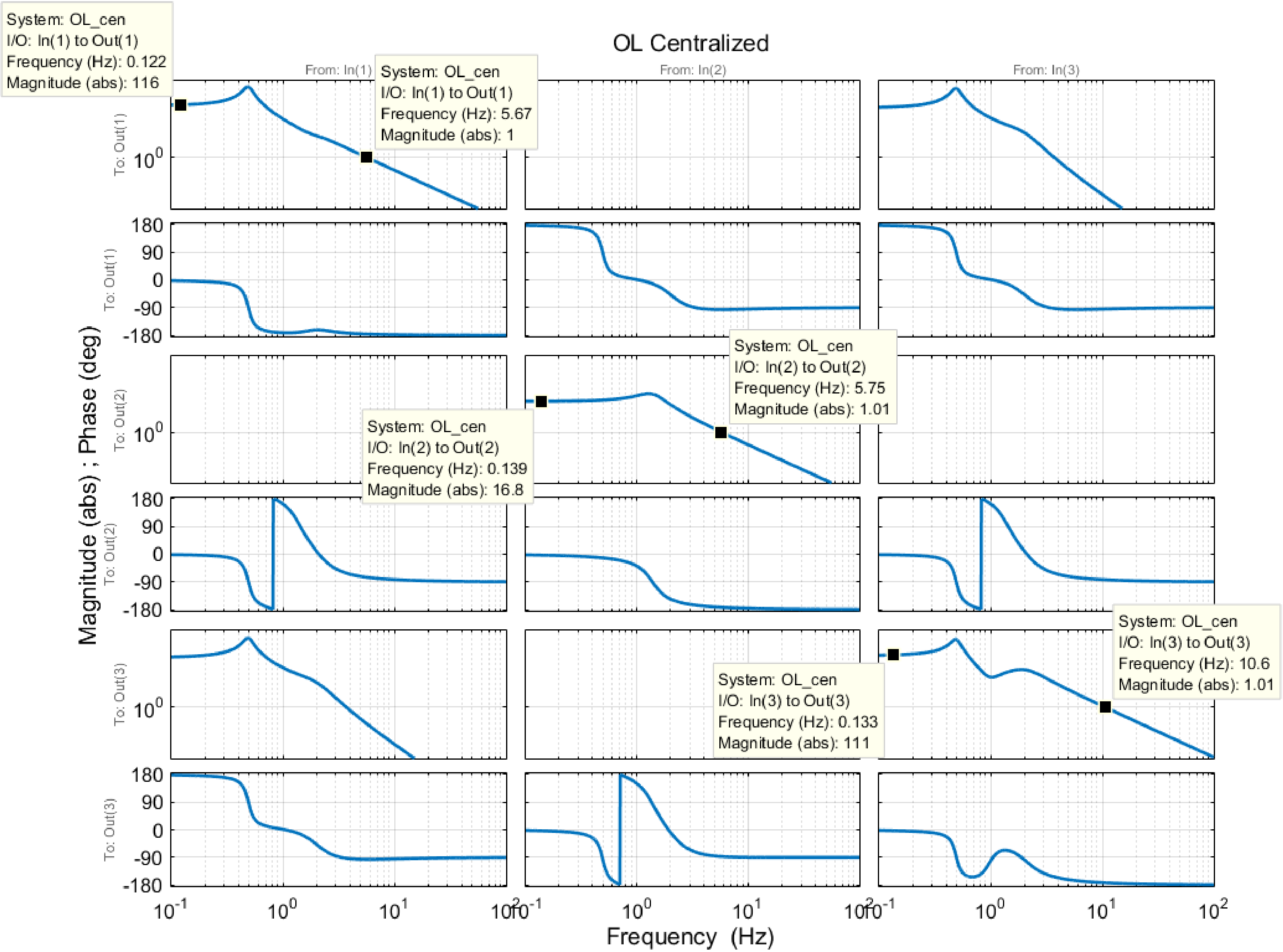
- When multiply it with the gain we are going to apply, we represent the open loop. We can verify on this plot if we have sufficient phase margin at the crossover frequency. If not, we will design the adequate controller.

- Here the gain we choose is 5.10^5: from previous experiments, a gain of 5.10^5 led to an isolation of +/- 100 and such gain could be experimentaly applied.

  Let's see if this gain leads to the same order of magnitude of performance here.

Centralized control

Design of the controller
Plot the plant*gain



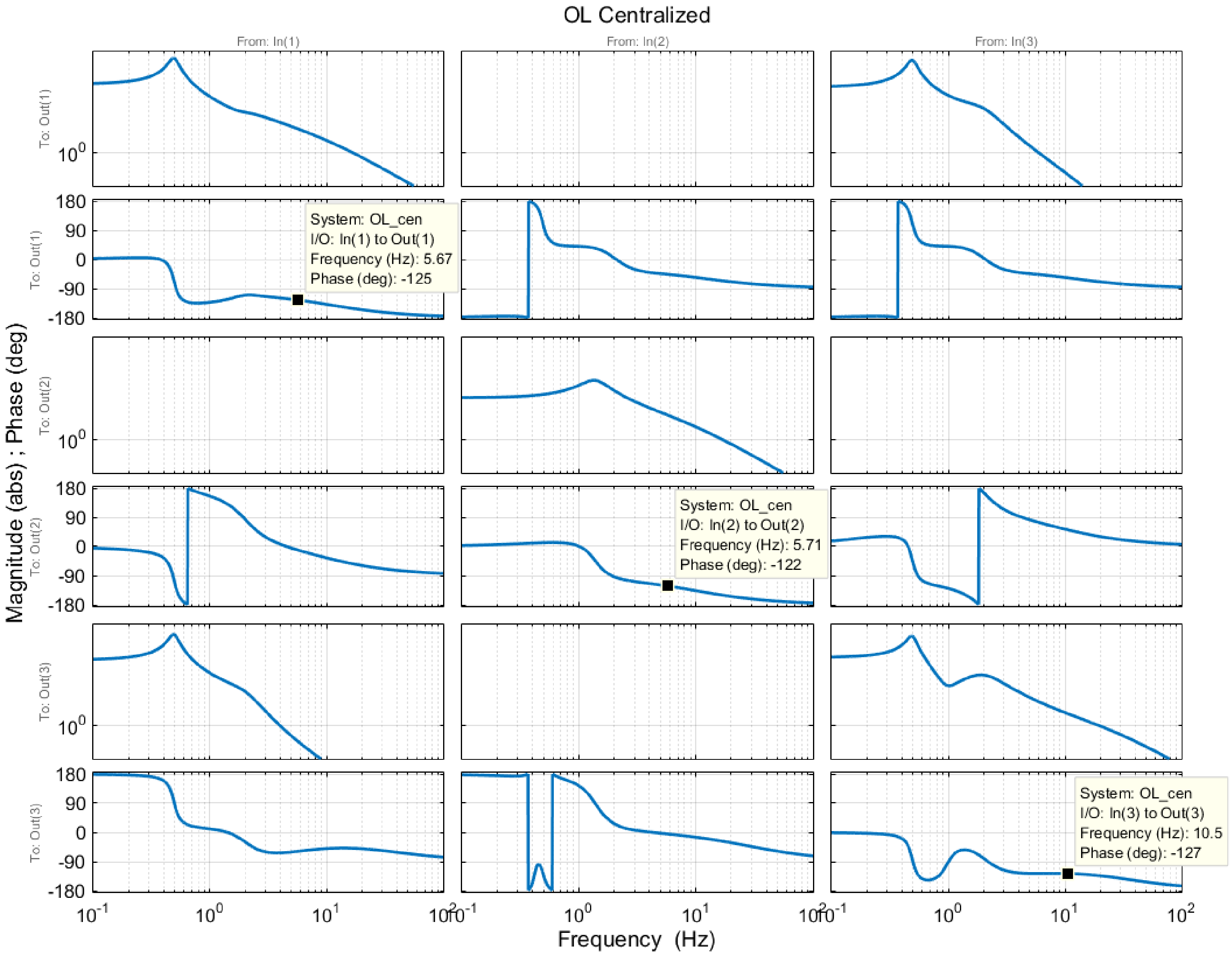
Here we show $g\, J_s^{-1}\, G\, (J_a^T)^{-1}$
Where g is the gain, J_s^{-1} is the inverse of the Jacobian of the sensors, G is the plant and $(J_a^T)^{-1}$ is the transpose inverse of the Jacobian of the actuators.

In other words, we show here the plant in the global coordinates, i.e. we show the x, z and θ motion (respectively row 1, 2 and 3) depending on the forces F_x, F_z and F_θ (respectively columns 1, 2 and 3).

We can see that at the crossover frequency, we have no phase margin for all diagonal elements → we have to design a lead for each element.

Design of the controller

Plot the plant*gain



The lead designed is

$$H = \begin{bmatrix} 10\dfrac{s+2\pi}{s+2\pi 10} & 0 & 0 \\ 0 & 10\dfrac{s+2\pi}{s+2\pi 10} & 0 \\ 0 & 0 & 6\dfrac{s+2\pi 5}{s+2\pi 30} \end{bmatrix}$$

On this graph, we have plotted $g\ H\ J_s^{-1}\ G\ (J_a^T)^{-1}$ and we can see that there is now enough phase margin.

NB: In Matlab, to generate a transfer function, we prefer to use the *zpk* function instead of the *tf* function as we have encountered some stability issues with the use of the *tf* function. For example, the upper left term is *zpk(-2*pi,-2*pi*10,10)*

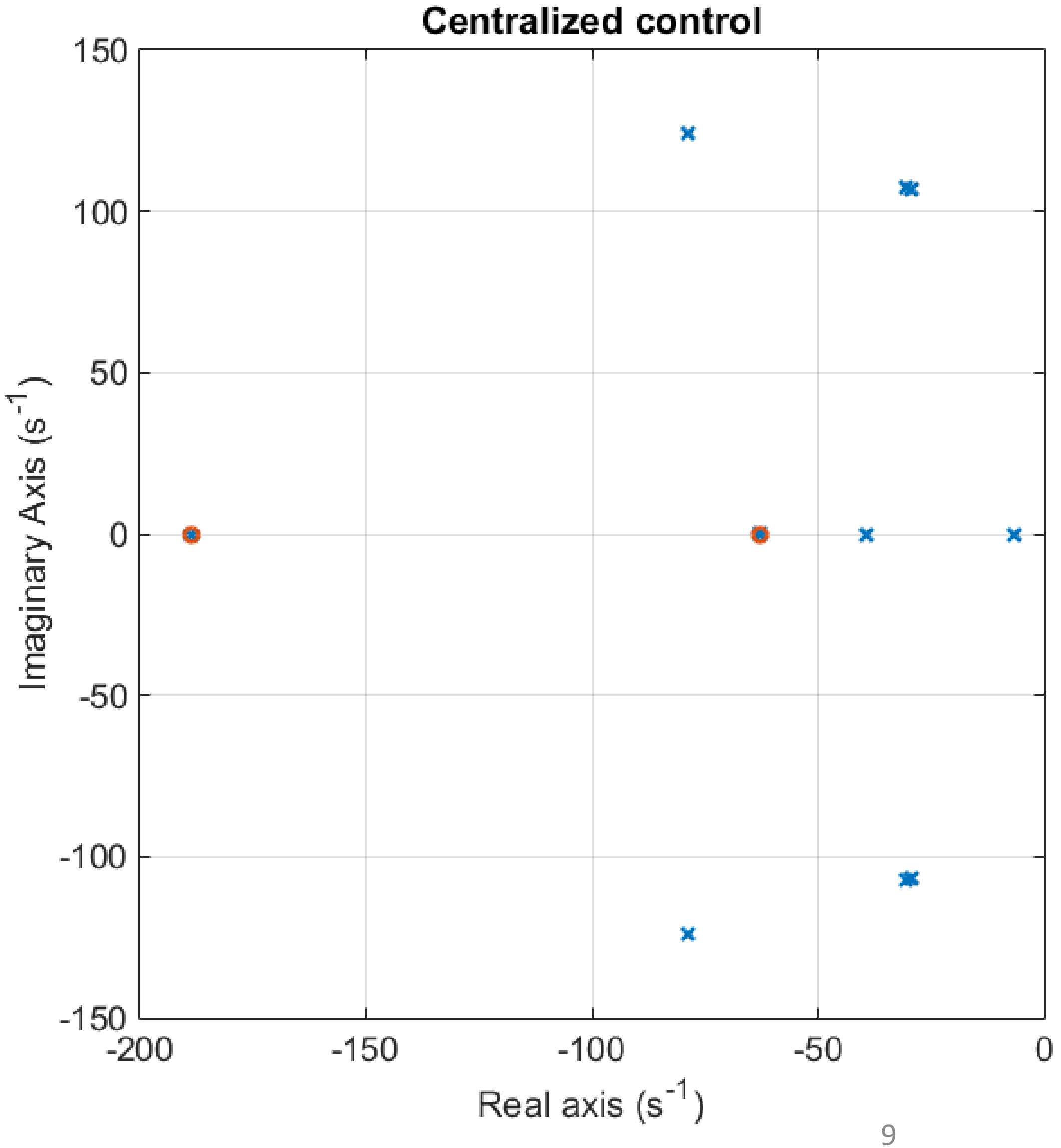
Design of the controller
Verify the poles and zeros location

Once the phase margins are sufficient, we can close the loop.

```
CL_cen =
feedback(system_dec,g*pinv(Jta)*H_cen_OL*pinv([Js
1;Js2]),[1 2 3],[1 2 3 4]);
```

Where `pinv([Js1;Js2])` corresponds to J_s^{-1}, `pinv(Jta)` corresponds to $(J_a^T)^{-1}$

Before looking at the performance, one last step is to verify the location of the poles and zeros of the closed loop using the *pzmap* function. Here, all poles and zeros are on the left hand side of the imaginary axis so we are happy.



Decentralized control

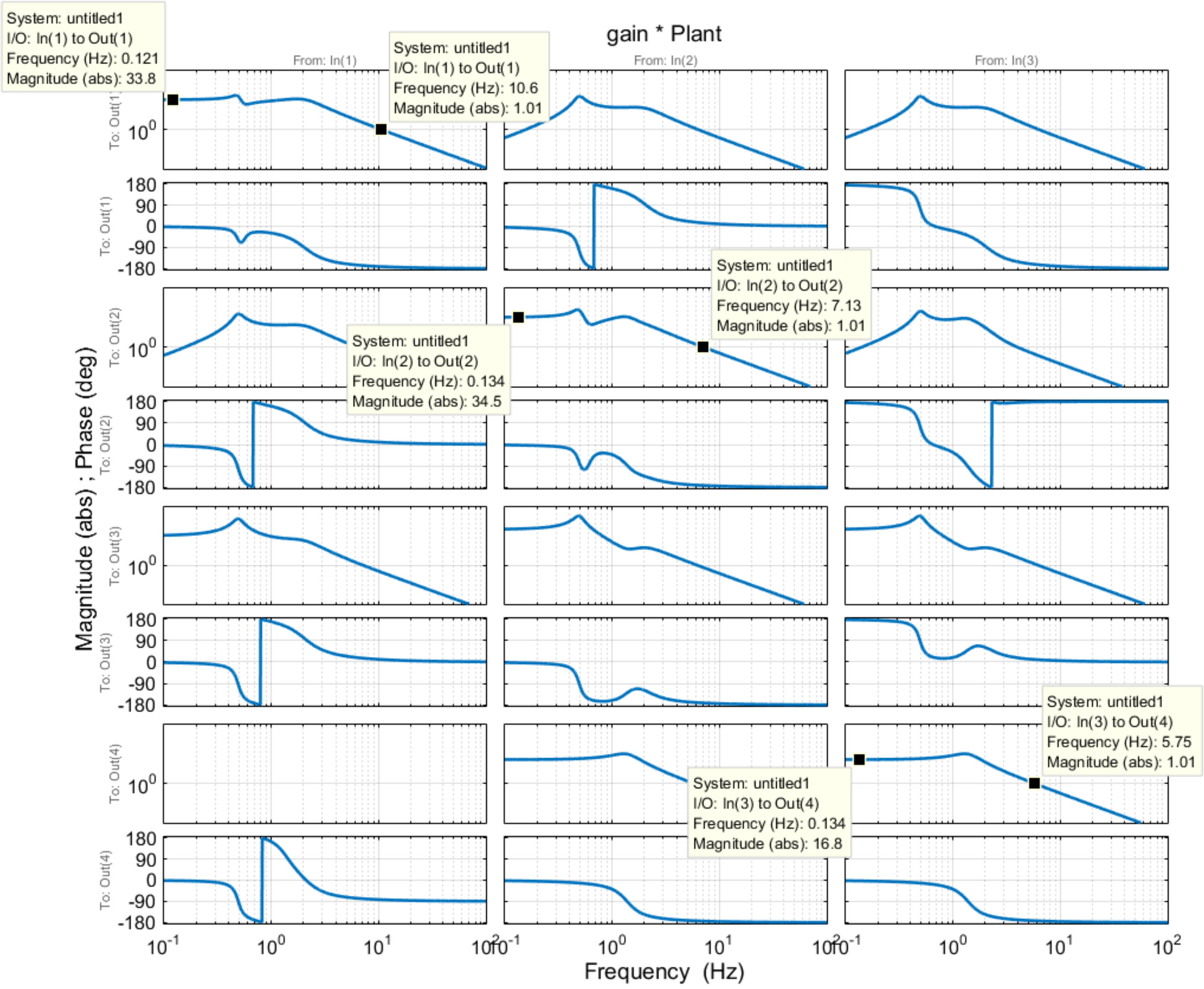
Design of the controller
Plot the plant*gain

There are four sensors and three actuators. Therefore, we can either use only three sensors or we can use a combination of the four sensors.

I would use only three sensors: one horizontal and the two vertical sensors as there are two vertical actuators and only one horizontal sensor. In my opinion, this solution will be easier to implement in practice.

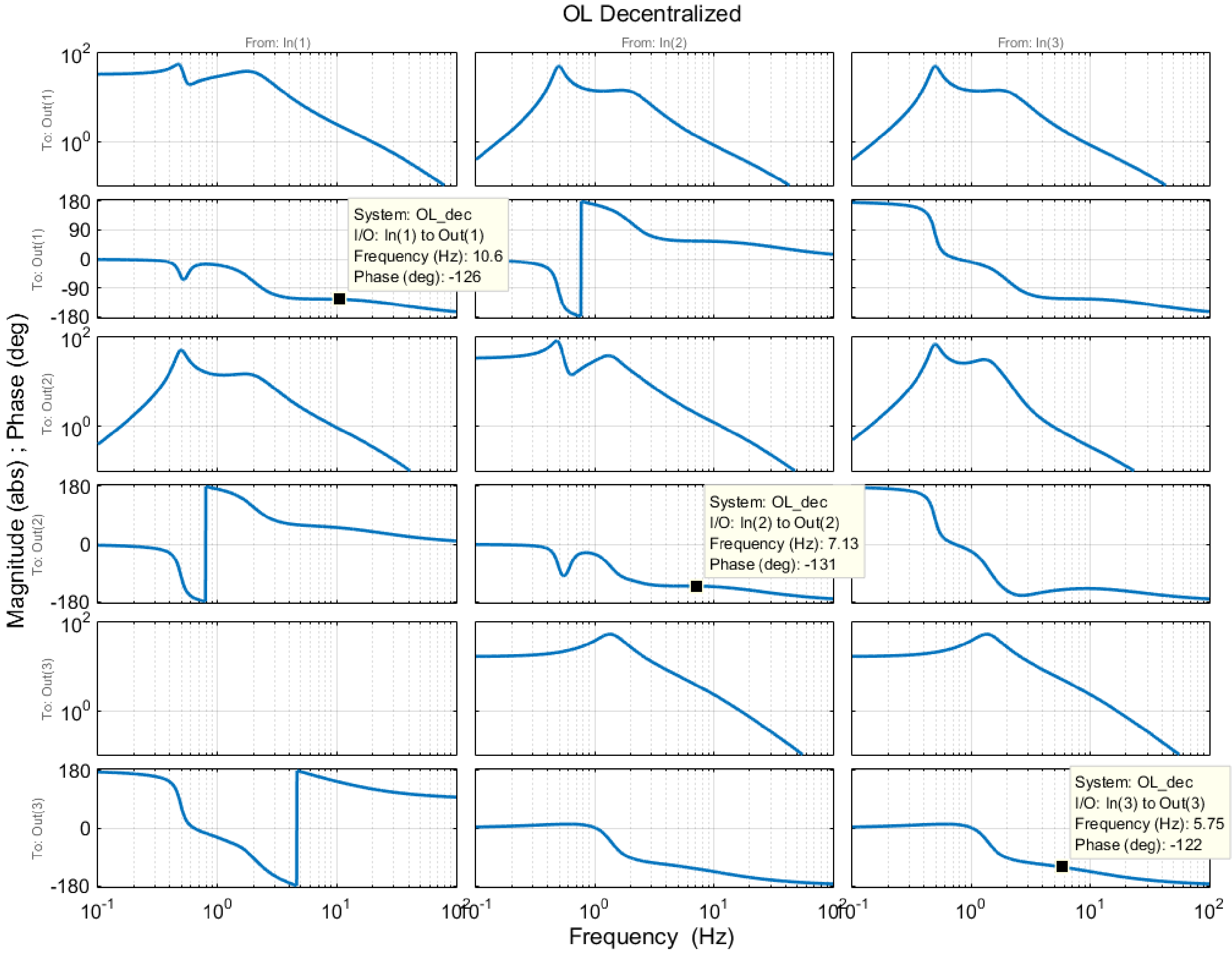
Design of the controller

Plot the plant*gain



The controller will be a 3x4 matrix whose third column will be only zeros. A lead has to be designed for the three sensors considered.

Design of the controller
Plot the plant*gain



The lead designed is

$$H = \begin{bmatrix} 6\dfrac{s+2\pi 5}{s+2\pi 30} & 0 & 0 \\ 0 & 5\dfrac{s+2\pi 4}{s+2\pi 20} & 0 \\ 0 & 0 & 10\dfrac{s+2\pi}{s+2\pi 10} \end{bmatrix}$$

On this graph, we have plotted $g\ H\ G$ and we can see that there is now enough phase margin.

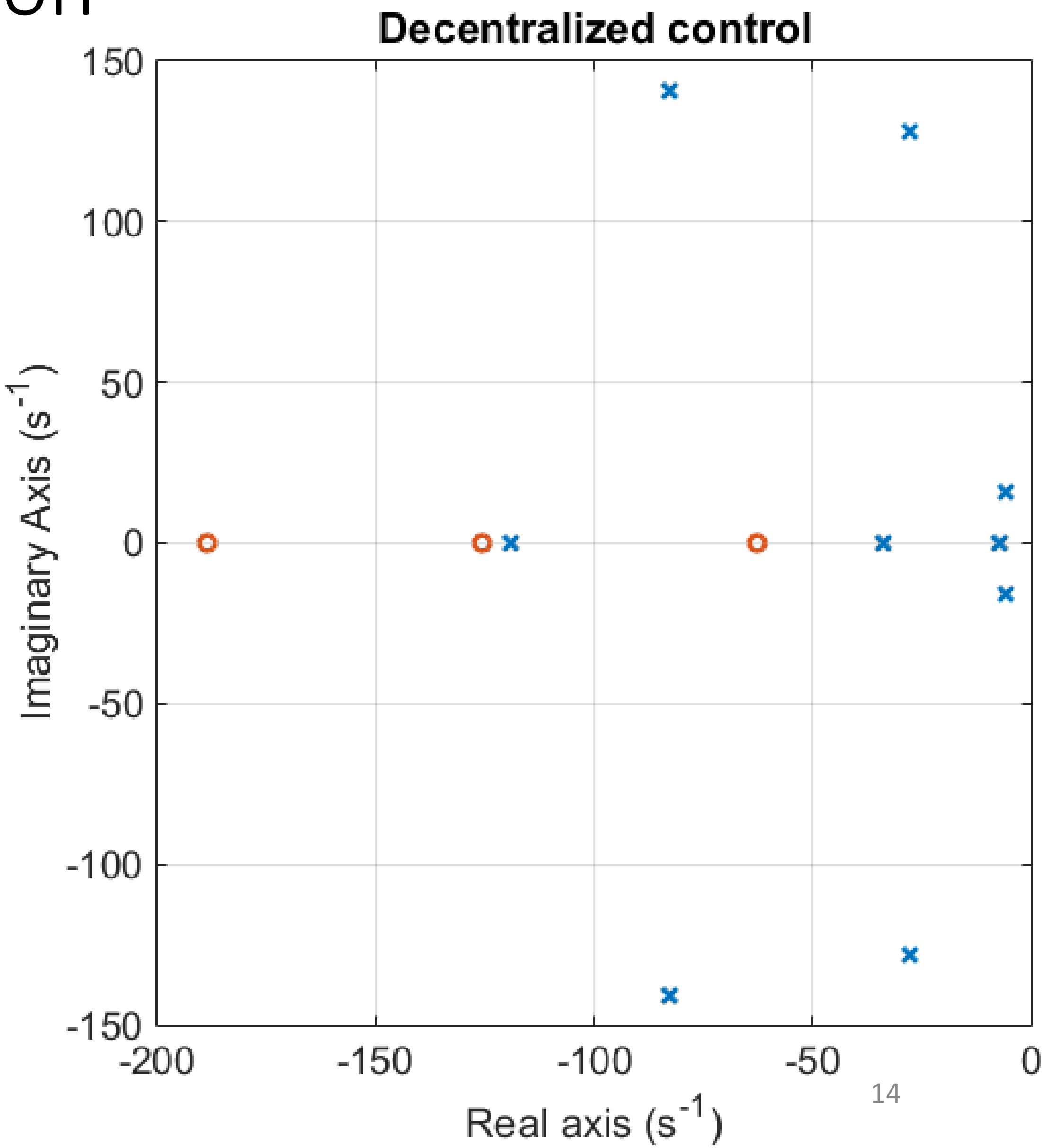
Design of the controller

Verify the poles and zeros location

Once the phase margins are sufficient, we can close the loop.

```
CL_dec = feedback(system_dec,g*H_dec,[1
2 3],[1 2 3 4]);
```

Before looking at the performance, one last step is to verify the location of the poles and zeros of the closed loop using the *pzmap* function. Here, all poles and zeros are on the left hand side of the imaginary axis so let's have a look at the performance now.



Performance

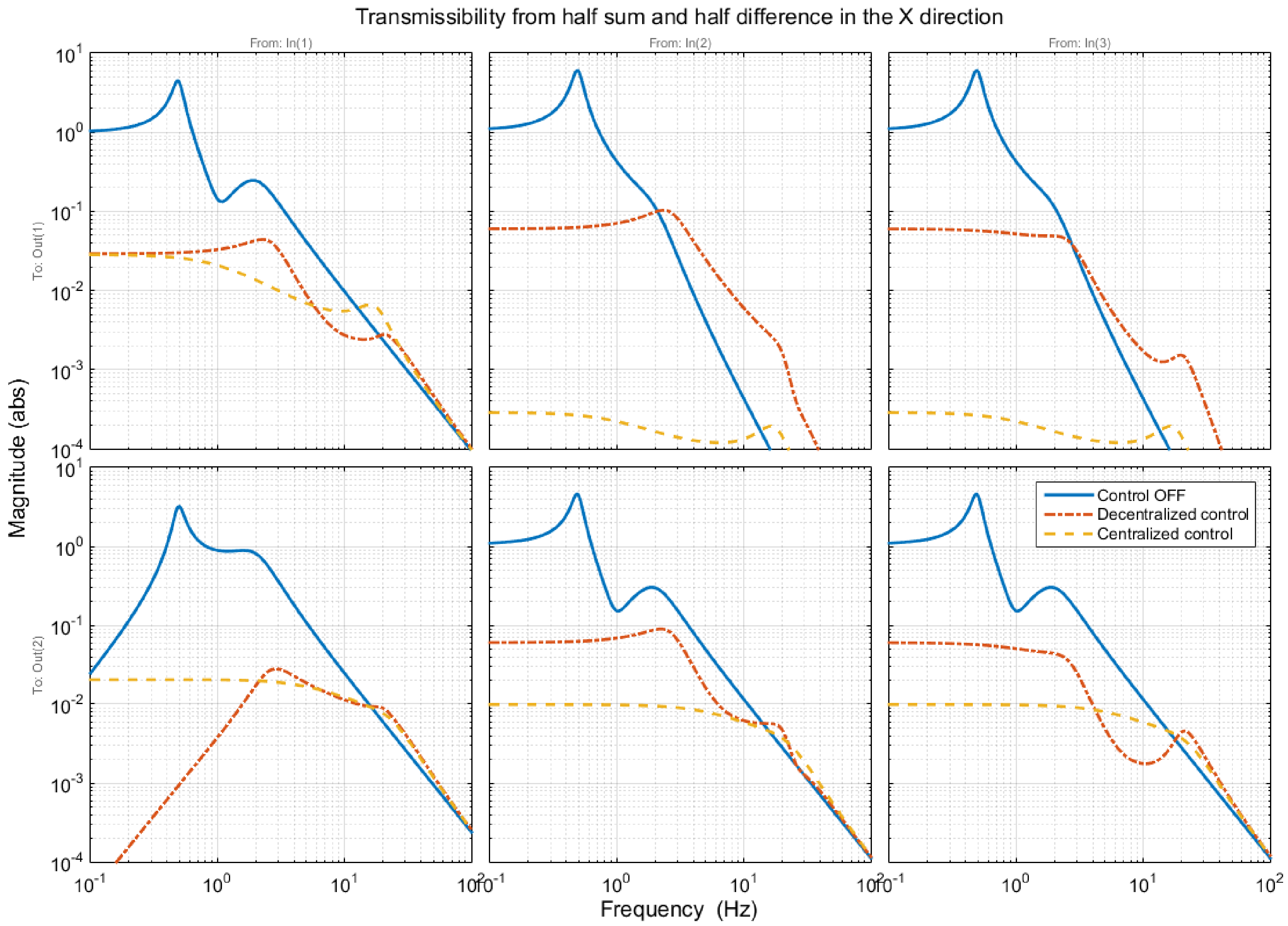
Transmissibility half sum and half difference horizontal direction

Half sum

$$\frac{x_1 + x_2}{2}$$

Half difference

$$\frac{x_1 - x_2}{2}$$



Horizontal ground motion

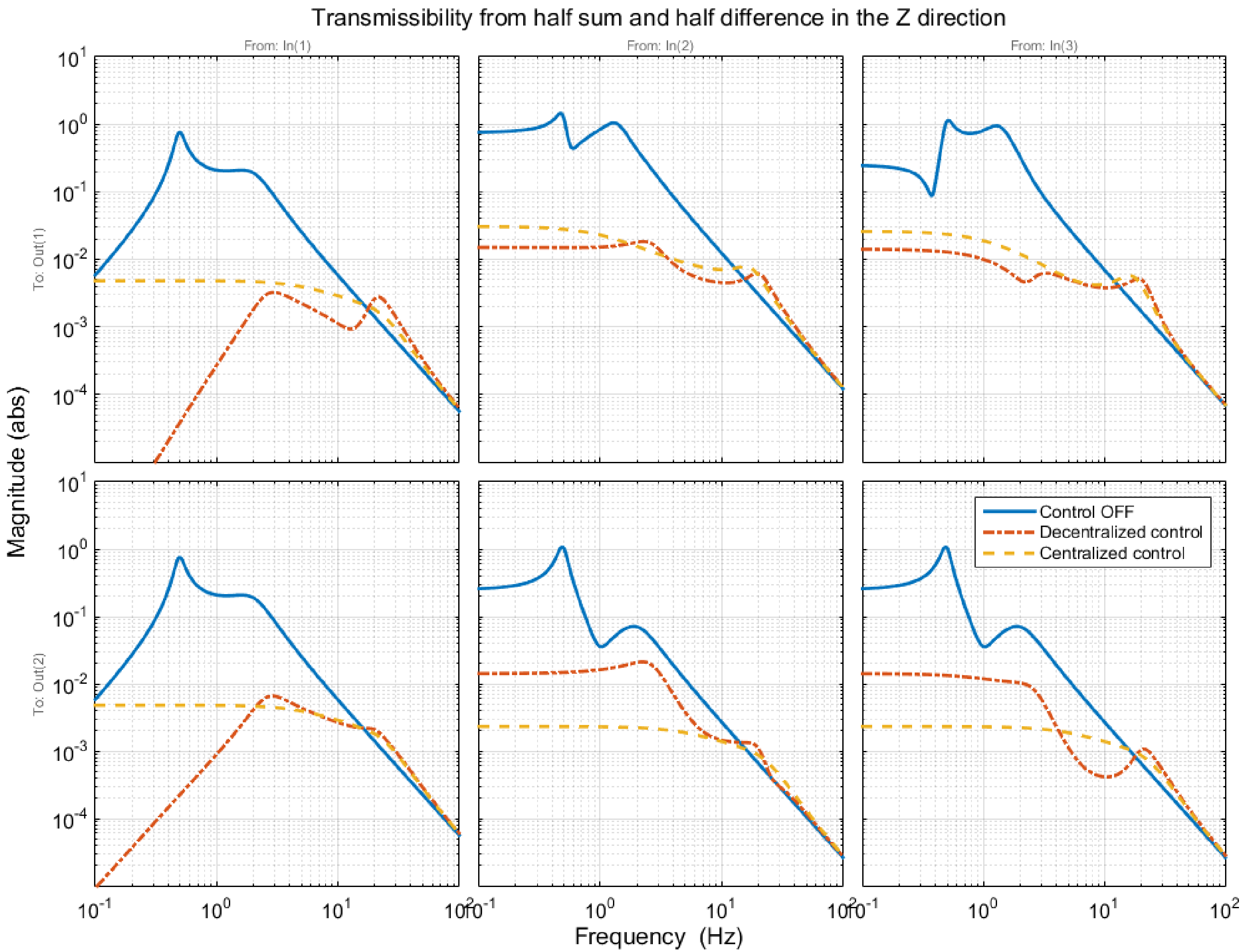
Vertical ground motion (left leg)

Vertical ground motion (right leg)

Transmissibility half sum and half difference vertical direction

Half sum
$$\frac{z_1 + z_2}{2}$$

Half difference
$$\frac{z_1 - z_2}{2}$$



Horizontal ground motion

Vertical ground motion (left leg)

Vertical ground motion (right leg)