

# **Flexible Joints - Test Bench**

Dehaeze Thomas

April 4, 2024

# Contents

- 1 Dimensional Measurements** **5**
- 1.1 Measurement Bench . . . . . 5
- 1.2 Measurement Results . . . . . 5
- 1.3 Bad flexible joints . . . . . 6
  
- 2 Development of the Measurement Test Bench** **7**
- 2.1 Measurement principle . . . . . 7
- 2.2 Developed test bench . . . . . 9
- 2.3 Error budget . . . . . 9
  
- 3 Bending Stiffness Measurement** **13**
- 3.1 Introduction . . . . . 13
- 3.2 Force Sensor Calibration . . . . . 15
- 3.3 Force Sensor Stiffness . . . . . 17
- 3.4 Analysis of one measurement . . . . . 17
- 3.5 Bending stiffness and bending stroke of all the flexible joints . . . . . 21
- 3.6 Analysis . . . . . 21
  
- 4 Conclusion** **24**

At both ends of the nano-hexapod struts, a flexible spherical joint is used. Ideally, these flexible joints would behave as perfect spherical joints, that is to say no bending and torsional stiffnesses, infinite shear and axial stiffnesses, unlimited bending and torsional stroke, no friction and no backlash.

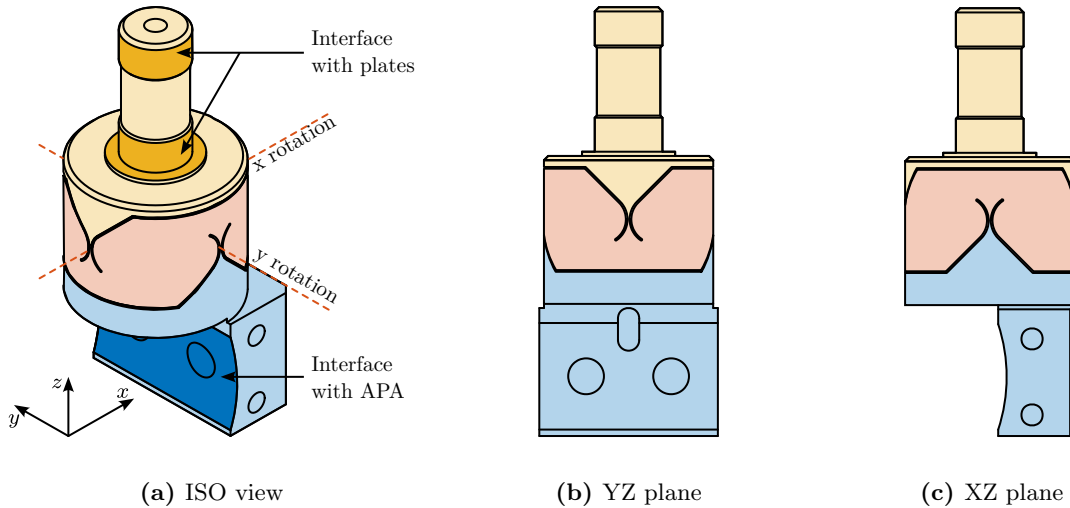
Deviations from this ideal properties will impact the dynamics of the Nano-Hexapod and could limit the attainable performances. During the detailed design phase, specifications in term of stiffness and stroke have been determined and are summarized in Table 1.

|                   | Specification   | FEM  |
|-------------------|-----------------|------|
| Axial Stiffness   | $> 100 N/\mu m$ | 94   |
| Shear Stiffness   | $> 1 N/\mu m$   | 13   |
| Bending Stiffness | $< 100 Nm/rad$  | 5    |
| Torsion Stiffness | $< 500 Nm/rad$  | 260  |
| Bending Stroke    | $> 1 mrad$      | 24.5 |
| Torsion Stroke    | $> 5 \mu rad$   |      |

**Table 1:** Specifications for the flexible joints and estimated characteristics from the Finite Element Model

After optimization using a finite element model, the geometry shown in Figure 1 has been obtained and the corresponding flexible joints characteristics are summarized in Table 1. This flexible joint is a monolithic piece of stainless steel<sup>1</sup> manufactured using wire electrical discharge machining. It serves several functions as shown in Figure 1a, such as:

- Rigid interfacing with the nano-hexapod plates (yellow surfaces)
- Rigid interfacing with the amplified piezoelectric actuator (blue surface)
- Allow two rotations between the “yellow” and the “blue” interfaces. The rotation axes are represented by the dashed lines which are intersecting

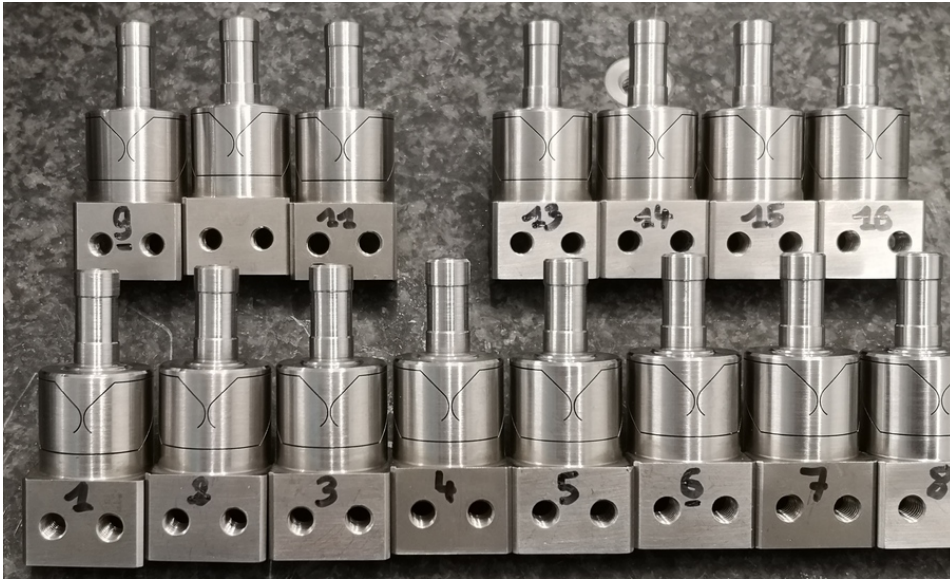


**Figure 1:** Geometry of the optimized flexible joints

16 flexible joints have been ordered (shown in Figure 2) such some selection can be made for the 12

<sup>1</sup>The alloy used is called *F16PH*, also refereed as “1.4542”

that will be used on the nano-hexapod.



**Figure 2:** 15 of the 16 flexible joints

In this document, the received flexible joints are characterized to make sure they are fulfilling the requirements and such that they can well be modelled.

First, the flexible joints are visually inspected, and the minimum gaps (responsible for most of the joint compliance) are measured (Section 1). Then, a test bench is developed to measure the bending stiffness of the flexible joints. The development of this test bench is presented in Section 2, including a noise budget and some requirements in terms of instrumentation. Finally, the test bench is manufactured and used to measure the bending stiffnesses of all the flexible joints. Results are shown in Section 3

| Sections  | Matlab File                                     |
|-----------|---|
| Section 1 | <code>test_joints_1_dim_meas.m</code>           |
| Section 2 | <code>test_joints_2_bench_dimensioning.m</code> |
| Section 3 | <code>test_joints_3_bending_stiff_meas.m</code> |

**Table 2:** Report sections and corresponding Matlab files

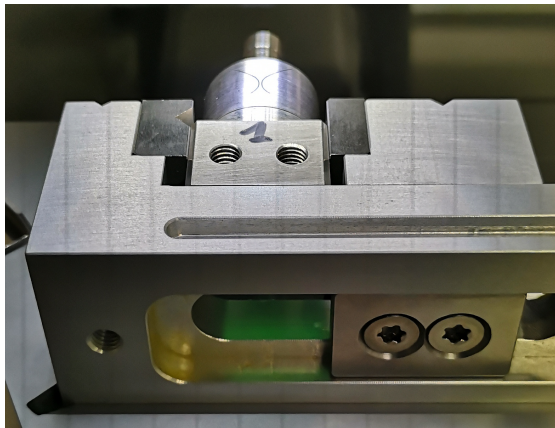
# 1 Dimensional Measurements

## 1.1 Measurement Bench

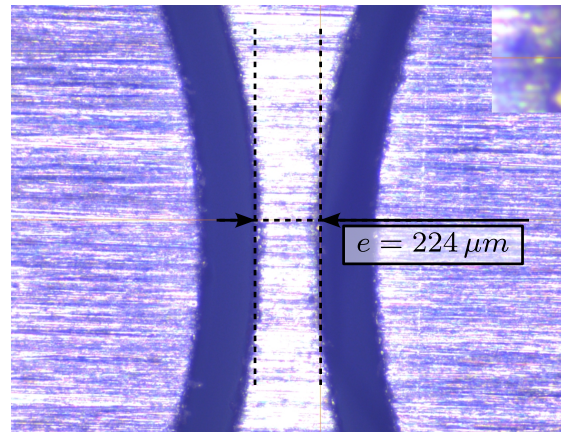
The dimensions of the flexible part in the Y-Z plane will contribute to the X-bending stiffness. Similarly, the dimensions of the flexible part in the X-Z plane will contribute to the Y-bending stiffness.

The setup to measure the dimension of the “X” flexible beam is shown in Figure 1.1b.

What we typically observe is shown in Figure 1.1a. It is then possible to estimate to dimension of the flexible beam with an accuracy of  $\approx 5 \mu m$ ,



(a) Flexible joint fixed on the profilometer



(b) Obtain image to estimate the gap

**Figure 1.1:** Setup to measure the dimension of the flexible beam corresponding to the X-bending stiffness. The flexible joint is fixed to the profilometer (a) and a image is obtained with which the gap can be estimated (b)

## 1.2 Measurement Results

The specified flexible beam thickness (gap) is  $250 \mu m$ . Four gaps are measured for each flexible joints (2 in the  $x$  direction and 2 in the  $y$  direction). The “beam thickness” is then estimated to be the mean between the gaps measured on opposite sides.

An histogram of the measured beam thicknesses is shown in Figure 1.2. The measured thickness is less than the specified value of  $250 \mu m$ , but this optical method may not be very accurate as the estimated gap can depend on the lighting of the part and of its proper alignment.

However, what is more important than the true value of the thickness is the consistency between all the flexible joints.

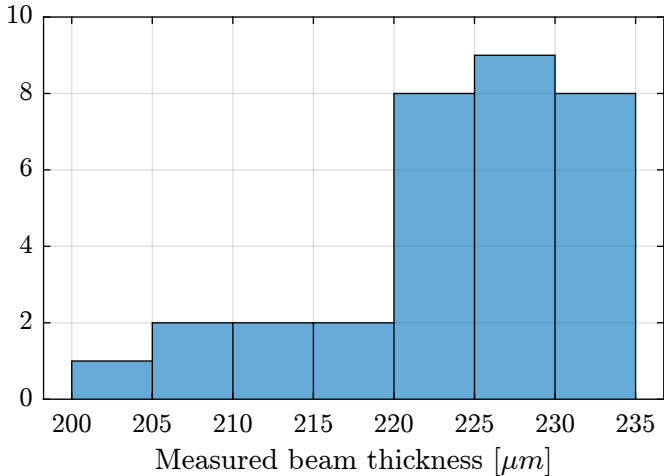
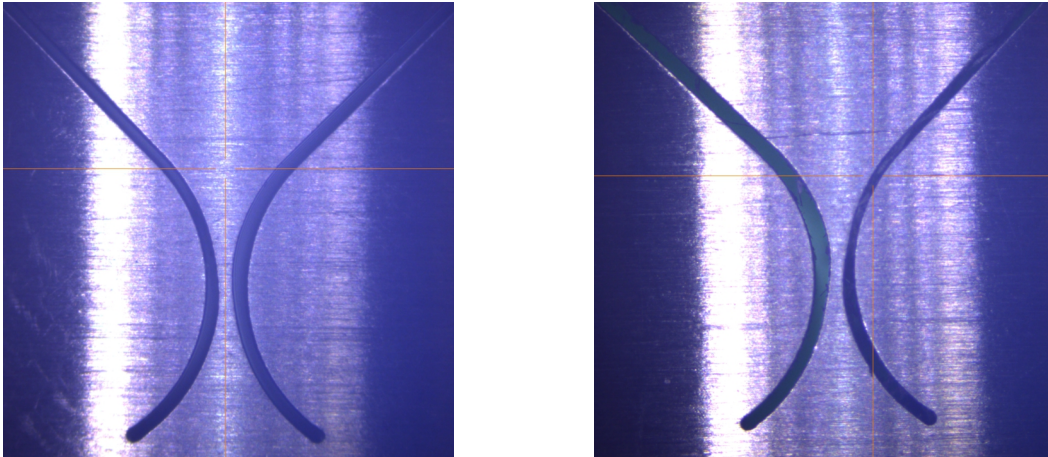


Figure 1.2: Histogram for the (16x2) measured beams' thickness

### 1.3 Bad flexible joints

Using this profilometer allowed to detect flexible joints with manufacturing defects such as non-symmetrical shape (see Figure 1.3a) or flexible joints with machining chips stuck in the gap (see Figure 1.3b).



(a) Non-Symmetrical shape

(b) "Chips" stuck in the air gap

Figure 1.3: Example of two flexible joints that were considered unsatisfactory after visual inspection

## 2 Development of the Measurement Test Bench

The most important characteristic of the flexible joint to be measured is its bending stiffness  $k_{R_x} \approx k_{R_y}$ .

To estimate the bending stiffness, the basic idea is to apply a torque  $T_x$  to the flexible joints and to measure its angular deflection  $\theta_x$ . Then, the bending stiffness can be computed from equation (2.1).

$$k_{R_x} = \frac{T_x}{\theta_x} \quad (2.1)$$

### 2.1 Measurement principle

As it is difficult to apply a pure torque, a “linear” force can be applied instead. The application point of the force should be far enough from the rotation axis such that the resulting bending motion is much larger than the displacement due to shear.

The working principle of the measurement bench is schematically shown in Figure 2.1. One part of the flexible joint is fixed. On the mobile part, a force  $F_x$  is applied which is equivalent to a torque applied on the flexible joint center. The induced rotation is measured with a displacement sensor  $d_x$ .

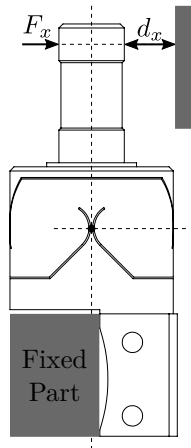
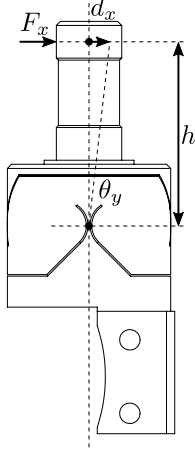


Figure 2.1: Test Bench - working principle

This test-bench will be used to have a first approximation of the bending stiffness and stroke of the flexible joints.



**Figure 2.2:** Geometry of the flexible joint

The flexible joint used for the Nano-Hexapod is shown in Figure 2.2. Its bending stiffness is foreseen to be  $k_{R_y} \approx 5 \frac{Nm}{rad}$  and its stroke  $\theta_{y,max} \approx 25 mrad$ .

The height between the flexible point (center of the joint) and the point where external forces are applied is  $h = 20 mm$ .

Let's define the parameters on Matlab.

**Required external applied force** The bending  $\theta_y$  of the flexible joint due to the force  $F_x$  is:

$$\theta_y = \frac{M_y}{k_{R_y}} = \frac{F_x h}{k_{R_y}} \quad (2.2)$$

Therefore, the applied force to test the full range of the flexible joint is:

$$F_{x,max} = \frac{k_{R_y} \theta_{y,max}}{h} \quad (2.3)$$

And we obtain:

$$F_{x,max} = 6.2 [N] \quad (2.4)$$

The measurement range of the force sensor should then be higher than  $6.2 N$ .

**Required actuator stroke and sensors range** The flexible joint is designed to allow a bending motion of  $\pm 25 mrad$ . The corresponding stroke at the location of the force sensor is:

$$d_{x,max} = h \tan(\theta_{y,max})$$

$$d_{max} = 0.5 [mm] \quad (2.5)$$



In order to test the full range of the flexible joint, the stroke of the translation stage used to move the force sensor should be higher than  $0.5\text{ mm}$ . Similarly, the measurement range of the displacement sensor should also be higher than  $0.5\text{ mm}$ .

## 2.2 Developed test bench

A CAD view of the measurement bench is shown in Figure 2.3a.

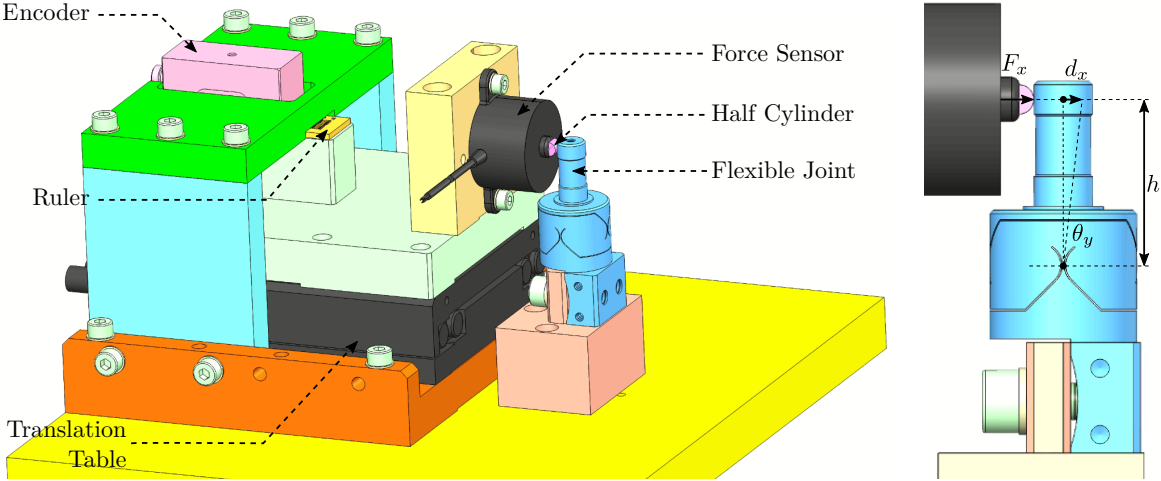
**Note**

Here are the different elements used in this bench:

- **Translation Stage:** V-408
- **Load Cells:** FC2231-0000-0010-L
- **Encoder:** Renishaw Resolute 1nm

Both the measured force and displacement are acquired at the same time using a Speedgoat machine. Explain why the encoder is here: in line with the measurement, no “abbe errors”

A side view of the bench with the important quantities are shown in Figure 2.3b.



(a) Schematic of the test bench to measure the bending stiffness of the flexible joints

(b) Zoom

**Figure 2.3:** Caption with reference to sub figure (??)

## 2.3 Error budget

Many things can impact the accuracy of the measured bending stiffness such as:

- Errors in the force and displacement measurement

- Shear effects
- Deflection of the Force sensor
- Errors in the geometry of the bench

In this section, we wish to estimate the attainable accuracy with the current bench, and identified the limiting factors.

**Finite Element Model** From the Finite Element Model, the stiffness and stroke of the flexible joint have been computed and summarized in Tables 2.1 and 2.2.

|       | Stiffness [N/um] | Max Force [N] | Stroke [um] |
|-------|------------------|---------------|-------------|
| Axial | 94               | 469           | 5           |
| Shear | 13               | 242           | 19          |

**Table 2.1:** Axial/Shear characteristics

|           | Stiffness [Nm/rad] | Max Torque [Nmm] | Stroke [mrad] |
|-----------|--------------------|------------------|---------------|
| Bending   | 5                  | 118              | 24            |
| Torsional | 260                | 1508             | 6             |

**Table 2.2:** Bending/Torsion characteristics

**Setup** The setup is schematically represented in Figure ??.

The force is applied on top of the flexible joint with a distance  $h$  with the joint's center. The displacement of the flexible joint is also measured at the same height.

The height between the joint's center and the force application point is:

**Effect of Bending** The torque applied is:

$$M_y = F_x \cdot h \quad (2.6)$$

The flexible joint is experiencing a rotation  $\theta_y$  due to the torque  $M_y$ :

$$\theta_y = \frac{M_y}{k_{R_y}} = \frac{F_x \cdot h}{k_{R_y}} \quad (2.7)$$

This rotation is then measured by the displacement sensor. The measured displacement is:

$$D_b = h \tan(\theta_y) = h \tan\left(\frac{F_x \cdot h}{k_{R_y}}\right) \quad (2.8)$$

**Computation of the bending stiffness** From equation (2.8), we can compute the bending stiffness:

$$k_{R_y} = \frac{F_x \cdot h}{\tan^{-1}\left(\frac{D_b}{h}\right)} \quad (2.9)$$

For small displacement, we have

$$\boxed{k_{R_y} \approx h^2 \frac{F_x}{d_x}} \quad (2.10)$$

And therefore, to precisely measure  $k_{R_y}$ , we need to:

- precisely measure the motion  $d_x$
- precisely measure the applied force  $F_x$
- precisely know the height of the force application point  $h$

**Estimation error due to force and displacement sensors accuracy** The maximum error on the measured displacement with the encoder is 40 nm. This is quite negligible compared to the measurement range of 0.5 mm.

The accuracy of the force sensor is around 1% and therefore, we should expect to have an accuracy on the measured stiffness of at most 1%.

**Estimation error due to Shear** The effect of Shear on the measured displacement is simply:

$$D_s = \frac{F_x}{k_s} \quad (2.11)$$

The measured displacement will be the effect of shear + effect of bending

$$d_x = D_b + D_s = h \tan\left(\frac{F_x \cdot h}{k_{R_y}}\right) + \frac{F_x}{k_s} \approx F_x \left(\frac{h^2}{k_{R_y}} + \frac{1}{k_s}\right) \quad (2.12)$$

The estimated bending stiffness  $k_{\text{est}}$  will then be:

$$k_{\text{est}} = h^2 \frac{F_x}{d_x} \approx k_{R_y} \frac{1}{1 + \frac{k_{R_y}}{k_s h^2}} \quad (2.13)$$

|   |
|---|
| Results                                     |
| The measurement error due to Shear is 0.1 % |

**Estimation error due to force sensor compression** The measured displacement is not done directly at the joint's location. The force sensor compression will then induce an error on the joint's stiffness.

The force sensor stiffness  $k_F$  is estimated to be around:

The measured displacement will be the sum of the displacement induced by the bending and by the compression of the force sensor:

$$d_x = D_b + \frac{F_x}{k_F} = h \tan\left(\frac{F_x \cdot h}{k_{R_y}}\right) + \frac{F_x}{k_F} \approx F_x \left(\frac{h^2}{k_{R_y}} + \frac{1}{k_F}\right) \quad (2.14)$$

The estimated bending stiffness  $k_{\text{est}}$  will then be:

$$k_{\text{est}} = h^2 \frac{F_x}{d_x} \approx k_{R_y} \frac{1}{1 + \frac{k_{R_y}}{k_F h^2}} \quad (2.15)$$

**Estimation error due to height estimation error** Let's consider an error in the estimation of the height from the application of the force to the joint's center:

$$h_{\text{est}} = h(1 + \epsilon) \quad (2.16)$$

The computed bending stiffness will be:

$$k_{\text{est}} \approx h_{\text{est}}^2 \frac{F_x}{d_x} \quad (2.17)$$

And the stiffness estimation error is:

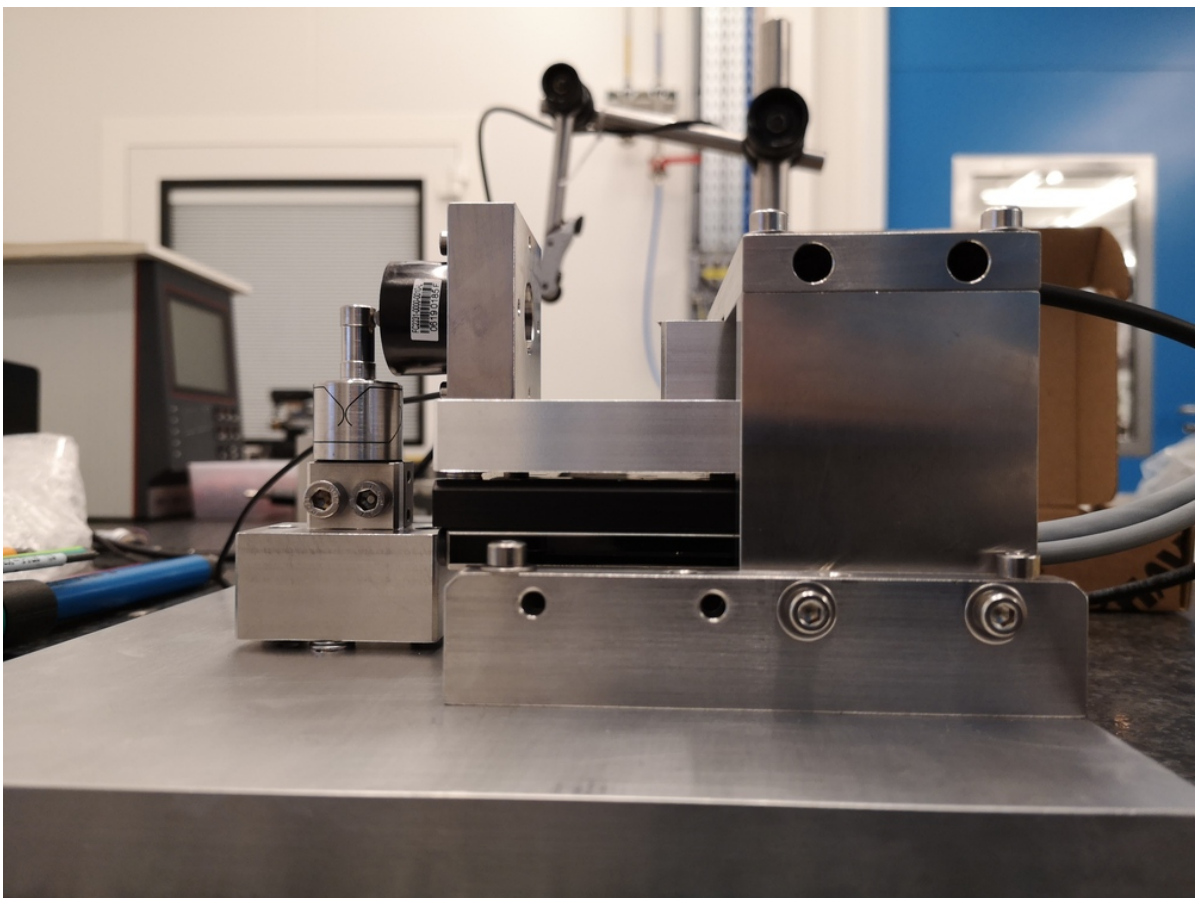
$$\frac{k_{\text{est}}}{k_{R_y}} = (1 + \epsilon)^2 \quad (2.18)$$

**Conclusion** Based on the above analysis, we should expect no better than few percent of accuracy using the current test-bench. This is well enough for a first estimation of the bending stiffness of the flexible joints.

## 3 Bending Stiffness Measurement

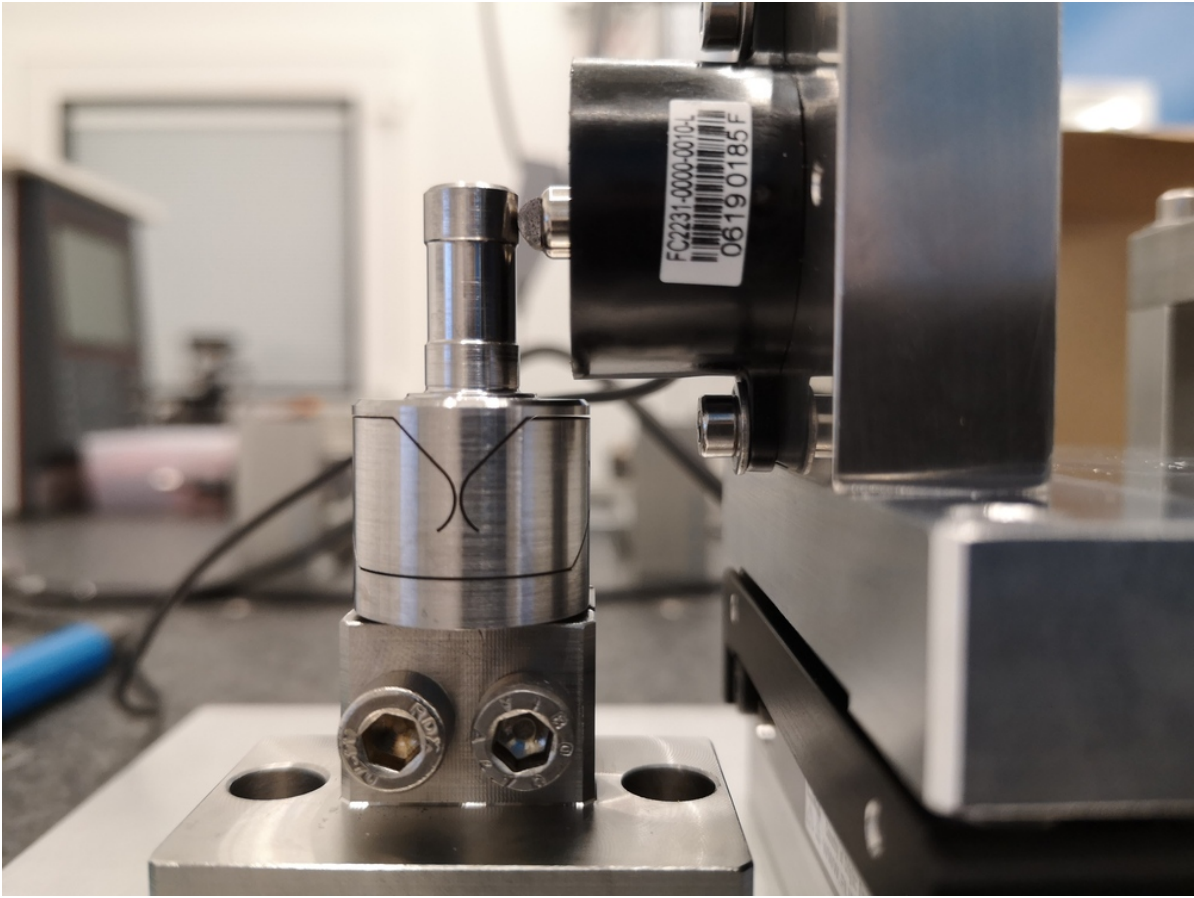
### 3.1 Introduction

A picture of the bench used to measure the X-bending stiffness of the flexible joints is shown in Figure 3.1. A closer view on flexible joint is shown in Figure 3.2 and a zoom on the force sensor tip is shown in Figure 3.3.

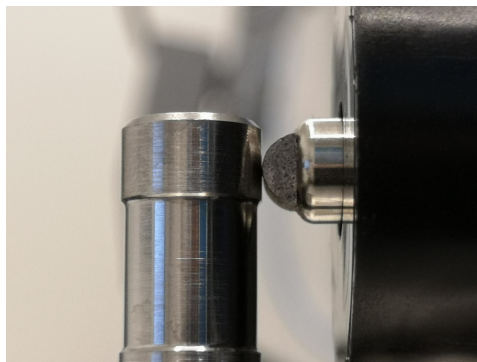


**Figure 3.1:** Side view of the flexible joint stiffness bench. X-Bending stiffness is measured.

The same bench used to measure the Y-bending stiffness of the flexible joint by pivoting the flexible joint by 90 degrees.



**Figure 3.2:** Zoom on the flexible joint - Side view



**Figure 3.3:** Zoom on the tip of the force sensor

## 3.2 Force Sensor Calibration

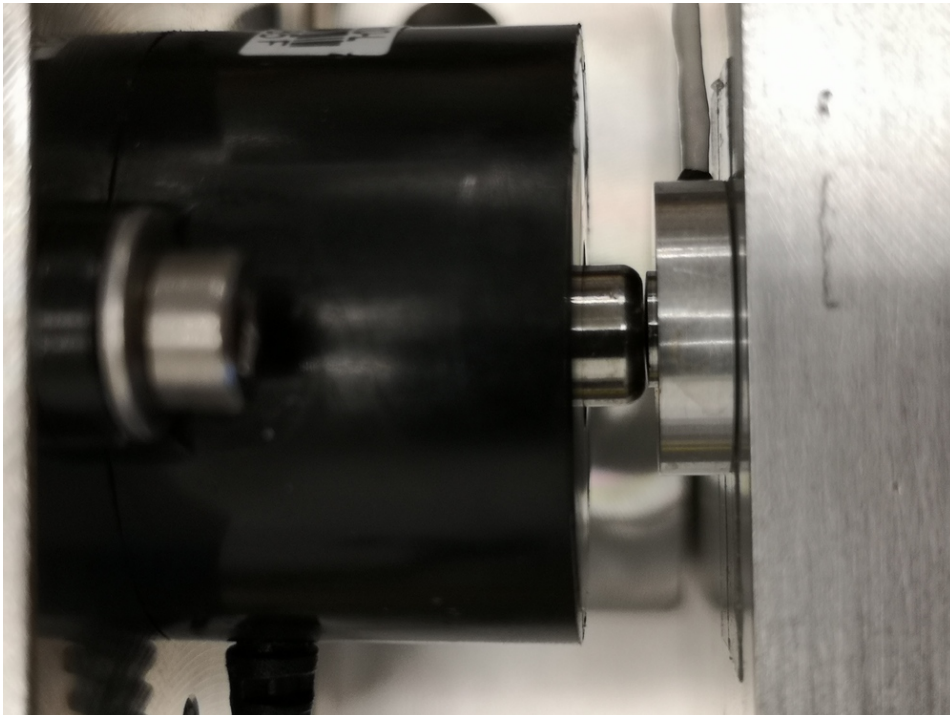
### Note

#### Load Cells:

- FC2231-0000-0010-L
- XFL212R

There are both specified to have  $\pm 1\%$  of non-linearity over the full range.

The XFL212R has a spherical interface while the FC2231 has a flat surface. Therefore, we should have a nice point contact when using the two force sensors as shown in Figure 3.4.



**Figure 3.4:** Zoom on the two force sensors in contact

The two force sensors are therefore measuring the exact same force, and we can compare the two measurements.

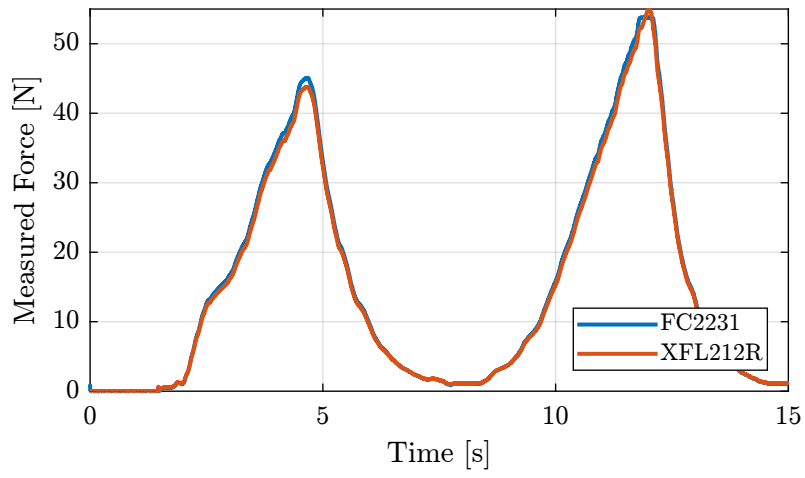
Let's load the measured force of both sensors. We remove any offset such that they are both measuring no force when not in contact.

Let's select only the first part from the moment they are in contact until the maximum force is reached.

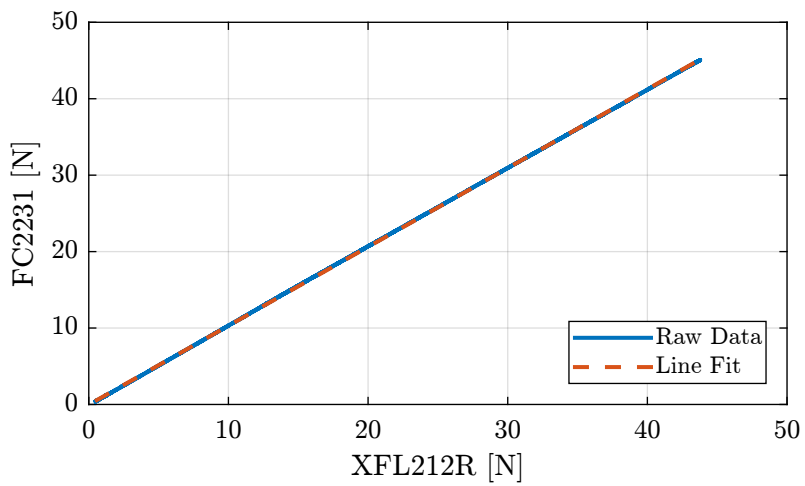
Then, let's make a linear fit between the two measured forces.

The two forces are plotted against each other as well as the linear fit in Figure 3.6.

The measurement error between the two sensors is shown in Figure 3.7. It is below 0.1N for the full



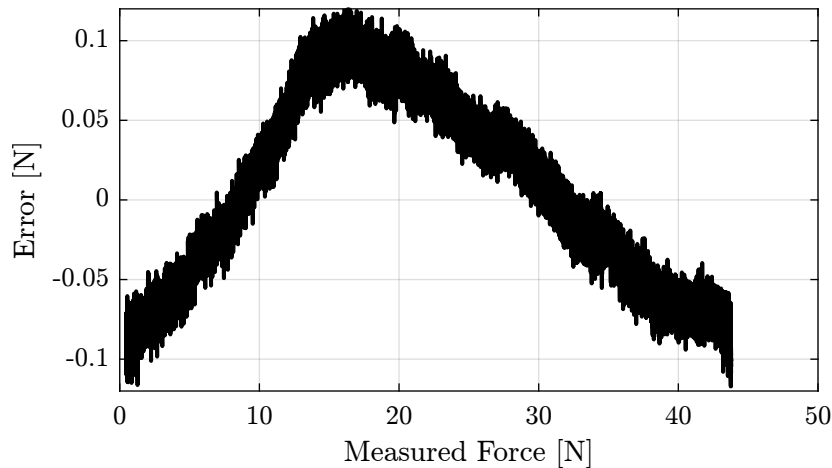
**Figure 3.5:** Measured force using both sensors as a function of time



**Figure 3.6:** Measured two forces and linear fit



measurement range.



**Figure 3.7:** Error in Newtons

### 3.3 Force Sensor Stiffness

The objective of this measurement is to estimate the stiffness of the force sensor [FC2231-0000-0010-L](#).

To do so, a very stiff element is fixed in front of the force sensor as shown in [Figure 3.8](#).

Then, we apply a force on the stiff element through the force sensor. We measure the deflection of the force sensor using an encoder.

Then, having the force and the deflection, we should be able to estimate the stiffness of the force sensor supposing the stiffness of the other elements are much larger.

From the documentation, the deflection of the sensor at the maximum load (50N) is 0.05mm, the stiffness is therefore foreseen to be around  $1 \text{ N}/\mu\text{m}$ .

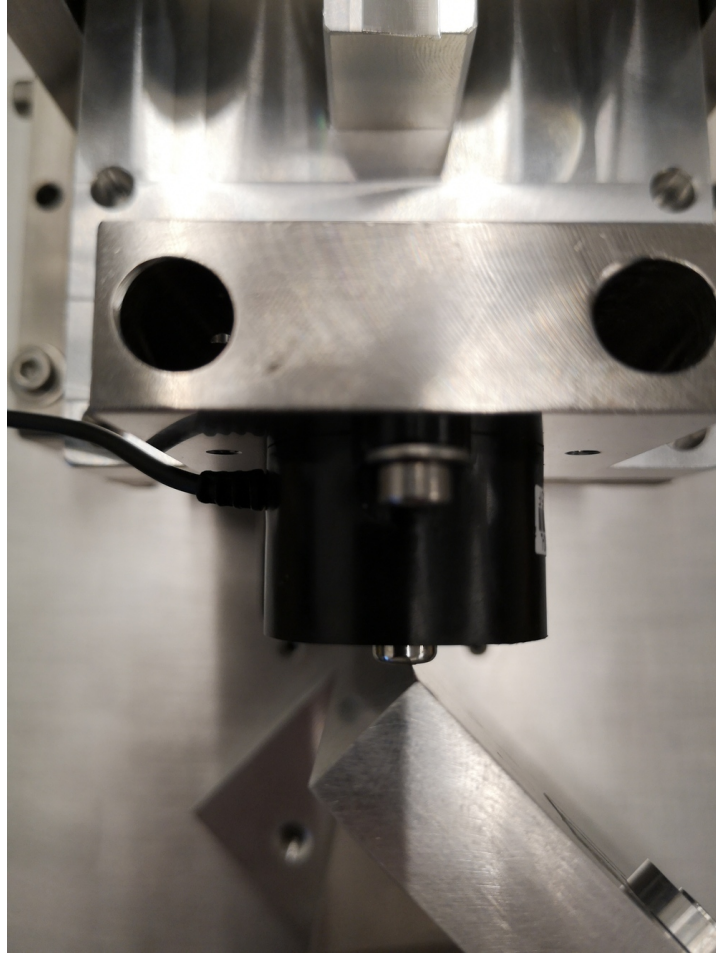
Let's load the measured force as well as the measured displacement. Some pre-processing is applied on the data. The linear fit is performed. The displacement as a function of the force as well as the linear fit are shown in [Figure 3.9](#).

And we obtain the following stiffness:

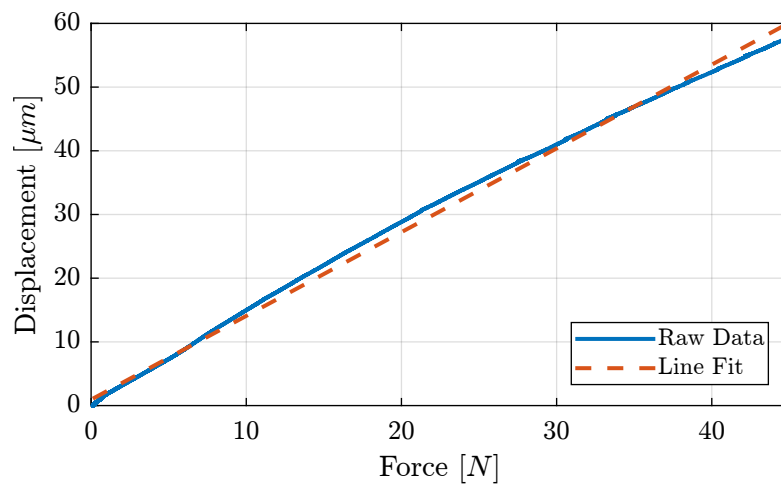
|                                    |
|------------------------------------|
| Results                            |
| $k = 0.76 \text{ [N}/\mu\text{m}]$ |

### 3.4 Analysis of one measurement

In this section is shown how the data are analysis in order to measured:



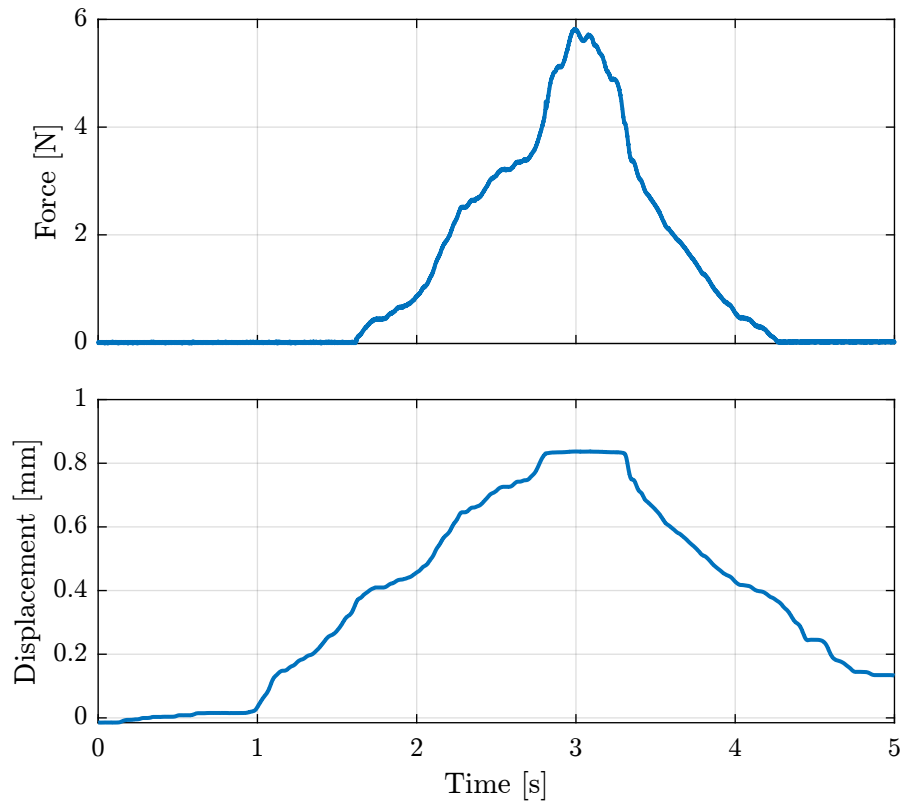
**Figure 3.8:** Bench used to measured the stiffness of the force sensor



**Figure 3.9:** Displacement as a function of the measured force

- the bending stiffness
- the bending stroke
- the stiffness once the mechanical stops are in contact

The height from the flexible joint's center and the point of application force  $h$  is defined below: The obtained time domain measurements are shown in Figure 3.10.



**Figure 3.10:** Typical time domain measurements

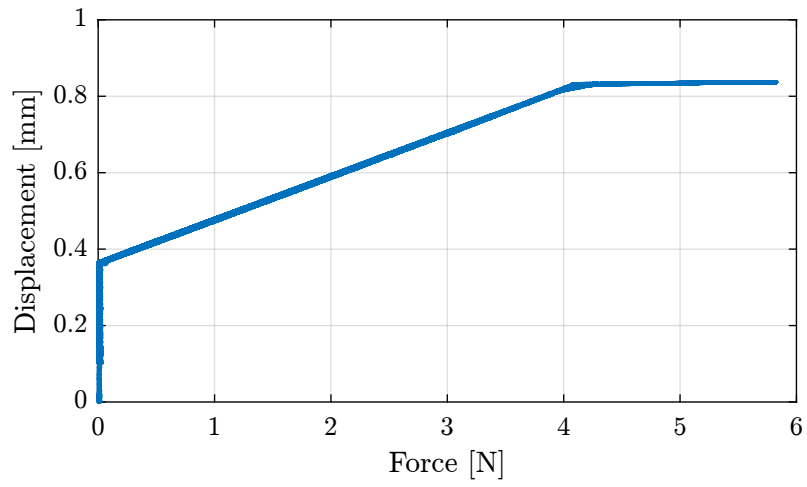
The displacement as a function of the force is then shown in Figure 3.11.

The bending stiffness can be estimated by computing the slope of the curve in Figure 3.11. The bending stroke and the stiffness when touching the mechanical stop can also be estimated from the same figure.

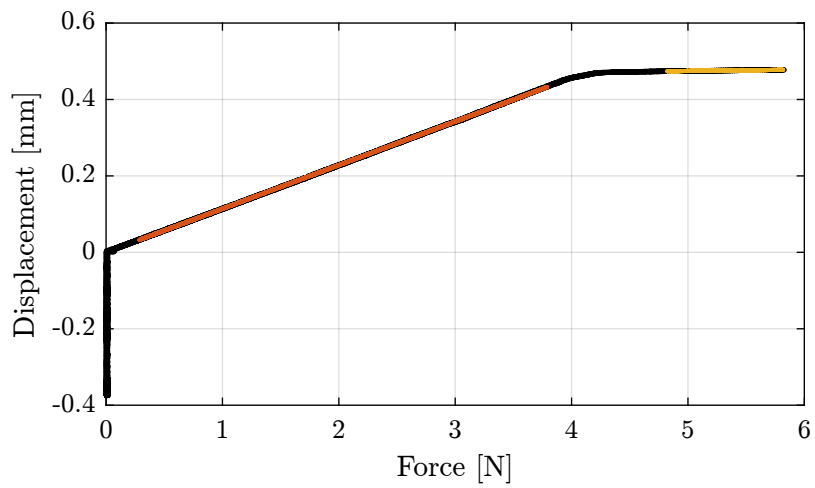
The raw data as well as the fit corresponding to the two stiffnesses are shown in Figure 3.12.

Then, the bending stroke is estimated as crossing point between the two fitted lines: The obtained characteristics are:

- Bending Stiffness: 5.5Nm/rad
- Bending Stiffness at stop: 173.6Nm/rad
- Bending Stroke: 18.9mrad



**Figure 3.11:** Typical measurement of the displacement as a function of the applied force



**Figure 3.12:** Typical measurement of the displacement as a function of the applied force with estimated linear fits

### 3.5 Bending stiffness and bending stroke of all the flexible joints

Now, let's estimate the bending stiffness and stroke for all the flexible joints.

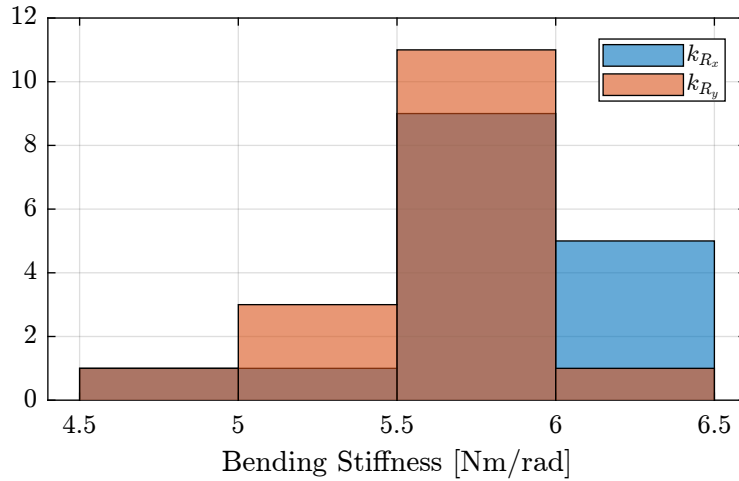
The results are summarized in Table 3.1 for the X direction and in Table 3.2 for the Y direction.

|    | $R_{R_x}$ [Nm/rad] | $k_{R_{x,s}}$ [Nm/rad] | $R_{x,max}$ [mrad] |
|----|--------------------|------------------------|--------------------|
| 1  | 5.5                | 173.6                  | 18.9               |
| 2  | 6.1                | 195.0                  | 17.6               |
| 3  | 6.1                | 191.3                  | 17.7               |
| 4  | 5.8                | 136.7                  | 18.3               |
| 5  | 5.7                | 88.9                   | 22.0               |
| 6  | 5.7                | 183.9                  | 18.7               |
| 7  | 5.7                | 157.9                  | 17.9               |
| 8  | 5.8                | 166.1                  | 17.9               |
| 9  | 5.8                | 159.5                  | 18.2               |
| 10 | 6.0                | 143.6                  | 18.1               |
| 11 | 5.0                | 163.8                  | 17.7               |
| 12 | 6.1                | 111.9                  | 17.0               |
| 13 | 6.0                | 142.0                  | 17.4               |
| 14 | 5.8                | 130.1                  | 17.9               |
| 15 | 5.7                | 170.7                  | 18.6               |
| 16 | 6.0                | 148.7                  | 17.5               |

**Table 3.1:** Measured characteristics of the flexible joints in the X direction

### 3.6 Analysis

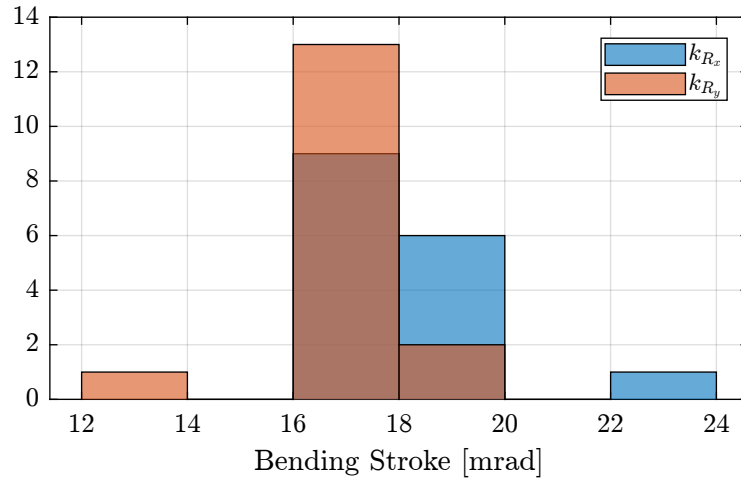
The dispersion of the measured bending stiffness is shown in Figure 3.13 and of the bending stroke in Figure 3.14.



**Figure 3.13:** Histogram of the measured bending stiffness

|    | $R_{R_y}$ [Nm/rad] | $k_{R_y,s}$ [Nm/rad] | $R_{y,max}$ [mrad] |
|----|--------------------|----------------------|--------------------|
| 1  | 5.7                | 323.5                | 17.9               |
| 2  | 5.9                | 306.0                | 17.2               |
| 3  | 6.0                | 224.4                | 16.8               |
| 4  | 5.7                | 247.3                | 17.8               |
| 5  | 5.8                | 250.9                | 13.0               |
| 6  | 5.8                | 244.5                | 17.8               |
| 7  | 5.3                | 214.8                | 18.1               |
| 8  | 5.8                | 217.2                | 17.6               |
| 9  | 5.7                | 225.0                | 17.6               |
| 10 | 6.0                | 254.7                | 17.3               |
| 11 | 4.9                | 261.1                | 18.4               |
| 12 | 5.9                | 161.5                | 16.7               |
| 13 | 6.1                | 227.6                | 16.8               |
| 14 | 5.9                | 221.3                | 17.8               |
| 15 | 5.4                | 241.5                | 17.8               |
| 16 | 5.3                | 291.1                | 17.7               |

**Table 3.2:** Measured characteristics of the flexible joints in the Y direction



**Figure 3.14:** Histogram of the measured bending stroke

The relation between the measured beam thickness and the measured bending stiffness is shown in Figure 3.15.

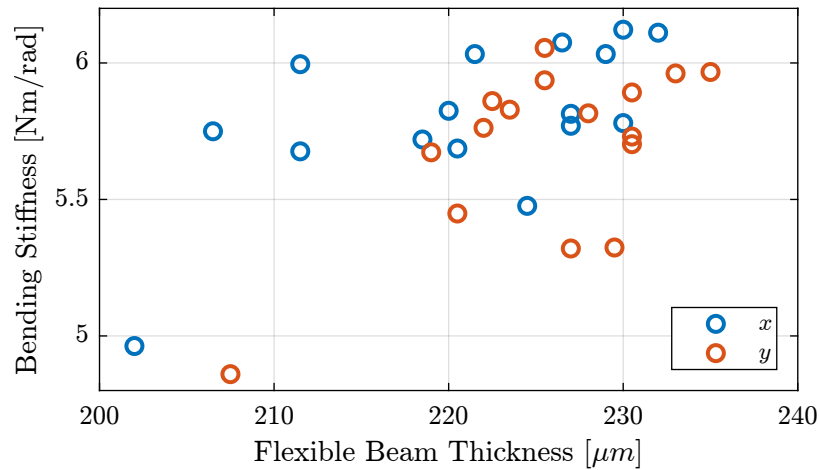


Figure 3.15: Measured bending stiffness as a function of the estimated flexible beam thickness

## Conclusion

### Important

The measured bending stiffness and bending stroke of the flexible joints are very close to the estimated one using a Finite Element Model.

The characteristics of all the flexible joints are also quite close to each other. This should allow us to model them with unique parameters.

## 4 Conclusion