

# **Flexible Joints - Test Bench**

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At both ends of the nano-hexapod struts, a flexible spherical joint is used. Ideally, these flexible joints would behave as perfect spherical joints, that is to say no bending and torsional stiffnesses, infinite shear and axial stiffnesses, unlimited bending and torsional stroke, no friction and no backlash.

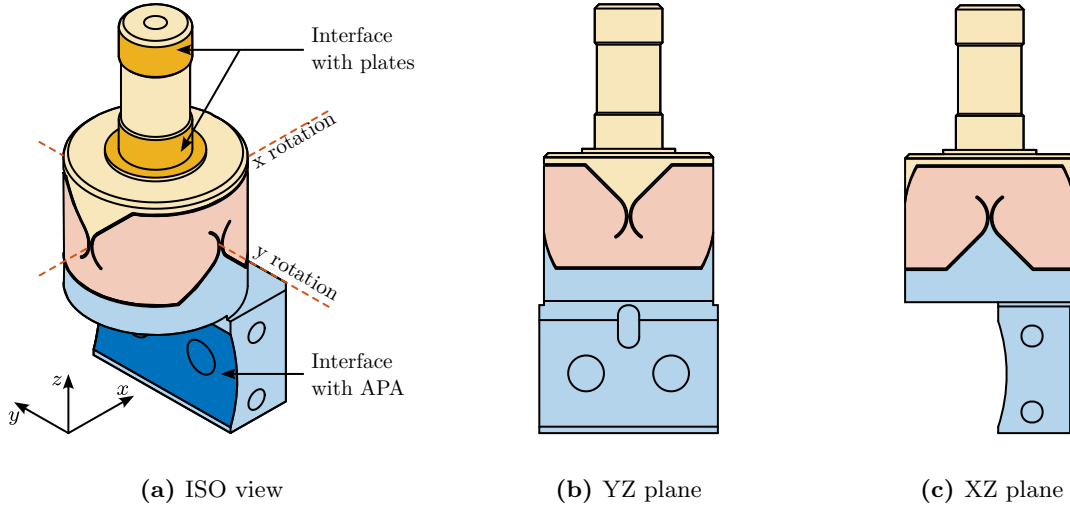
Deviations from this ideal properties will impact the dynamics of the Nano-Hexapod and could limit the attainable performances. During the detailed design phase, specifications in term of stiffness and stroke have been determined and are summarized in Table 1.

	Specification	FEM
Axial Stiffness	$> 100 N/\mu m$	94
Shear Stiffness	$> 1 N/\mu m$	13
Bending Stiffness	$< 100 Nm/rad$	5
Torsion Stiffness	$< 500 Nm/rad$	260
Bending Stroke	$> 1 mrad$	24.5

**Table 1:** Specifications for the flexible joints and estimated characteristics from the Finite Element Model

After optimization using a finite element model, the geometry shown in Figure 1 has been obtained and the corresponding flexible joints characteristics are summarized in Table 1. This flexible joint is a monolithic piece of stainless steel<sup>1</sup> manufactured using wire electrical discharge machining. It serves several functions as shown in Figure 1a, such as:

- Rigid interfacing with the nano-hexapod plates (yellow surfaces)
- Rigid interfacing with the amplified piezoelectric actuator (blue surface)
- Allow two rotations between the “yellow” and the “blue” interfaces. The rotation axes are represented by the dashed lines which are intersecting



**Figure 1:** Geometry of the optimized flexible joints

16 flexible joints have been ordered (shown in Figure 2a) such some selection can be made for the 12 that will be used on the nano-hexapod.

<sup>1</sup>The alloy used is called *F16PH*, also refereed as “1.4542”



(a) 15 of the 16 received flexible joints



(b) Zoom on one flexible joint

**Figure 2:** Pictures of the received 16 flexible joints

In this document, the received flexible joints are characterized to make sure they are fulfilling the requirements and such that they can well be modelled.

First, the flexible joints are visually inspected, and the minimum gaps (responsible for most of the joint compliance) are measured (Section 1). Then, a test bench is developed to measure the bending stiffness of the flexible joints. The development of this test bench is presented in Section 2, including a noise budget and some requirements in terms of instrumentation. Finally, the test bench is manufactured and used to measure the bending stiffnesses of all the flexible joints. Results are shown in Section 3

Sections	Matlab File
Section 1	test_joints_1_dim_meas.m
Section 2	test_joints_2_bench_dimensioning.m
Section 3	test_joints_3_bending_stiff_meas.m

**Table 2:** Report sections and corresponding Matlab files

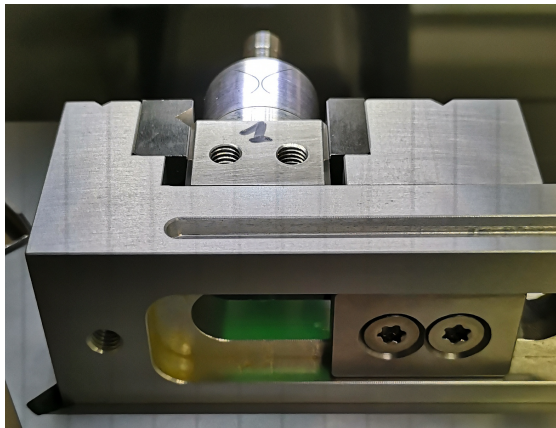
# 1 Dimensional Measurements

## 1.1 Measurement Bench

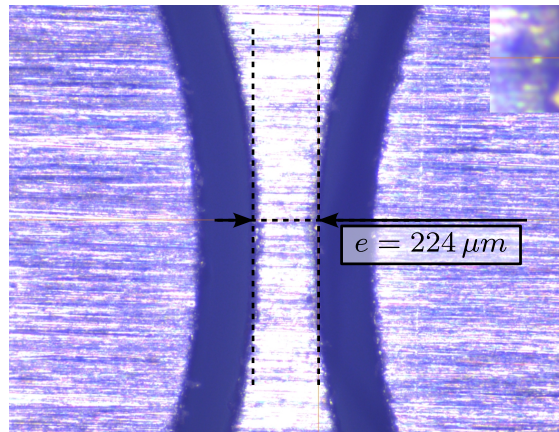
The dimensions of the flexible part in the Y-Z plane will contribute to the X-bending stiffness. Similarly, the dimensions of the flexible part in the X-Z plane will contribute to the Y-bending stiffness.

The setup to measure the dimension of the “X” flexible beam is shown in Figure 1.1b.

What we typically observe is shown in Figure 1.1a. It is then possible to estimate to dimension of the flexible beam with an accuracy of  $\approx 5 \mu m$ ,



(a) Flexible joint fixed on the profilometer



(b) Obtain image to estimate the gap

**Figure 1.1:** Setup to measure the dimension of the flexible beam corresponding to the X-bending stiffness. The flexible joint is fixed to the profilometer (a) and a image is obtained with which the gap can be estimated (b)

## 1.2 Measurement Results

The specified flexible beam thickness (gap) is  $250 \mu m$ . Four gaps are measured for each flexible joints (2 in the  $x$  direction and 2 in the  $y$  direction). The “beam thickness” is then estimated to be the mean between the gaps measured on opposite sides.

An histogram of the measured beam thicknesses is shown in Figure 1.2. The measured thickness is less than the specified value of  $250 \mu m$ , but this optical method may not be very accurate as the estimated gap can depend on the lighting of the part and of its proper alignment.

However, what is more important than the true value of the thickness is the consistency between all the flexible joints.

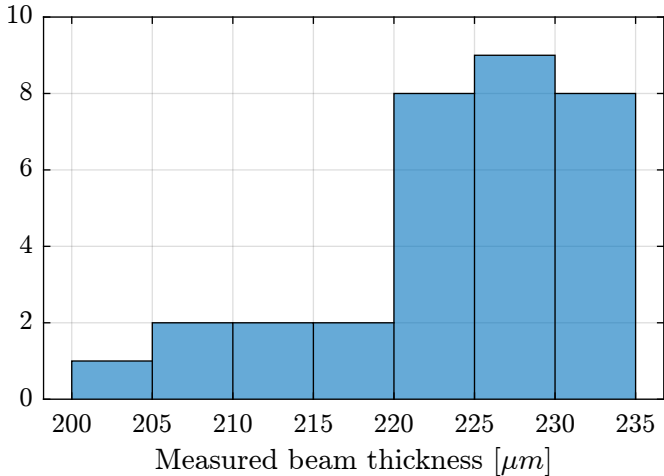


Figure 1.2: Histogram for the (16x2) measured beams' thickness

### 1.3 Bad flexible joints

Using this profilometer allowed to detect flexible joints with manufacturing defects such as non-symmetrical shape (see Figure 1.3a) or flexible joints with machining chips stuck in the gap (see Figure 1.3b).

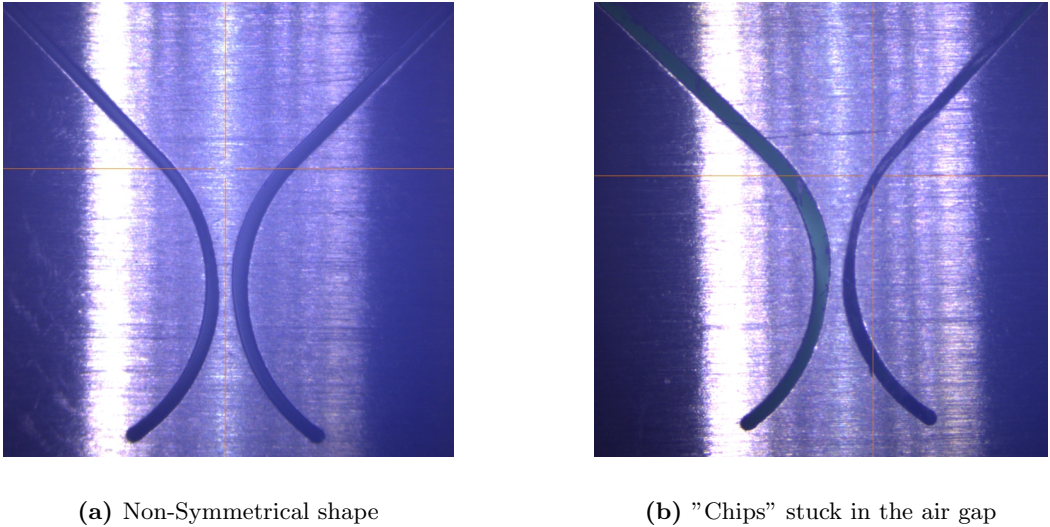


Figure 1.3: Example of two flexible joints that were considered unsatisfactory after visual inspection

## 2 Development of the Measurement Test Bench

The most important characteristic of the flexible joint to be measured is its bending stiffness  $k_{R_x} \approx k_{R_y}$ .

To estimate the bending stiffness, the basic idea is to apply a torque  $T_x$  to the flexible joints and to measure its angular deflection  $\theta_x$ . Then, the bending stiffness can be computed from equation (2.1).

$$\boxed{k_{R_x} = \frac{T_x}{\theta_x}, \quad k_{R_y} = \frac{T_y}{\theta_y}} \quad (2.1)$$

### 2.1 Measurement principle

**Torque and Rotation measurement** In order to apply torque  $T_y$  between the two mobile parts of the flexible joint, a known “linear” force  $F_x$  can be applied instead at the certain height  $h$  with respect to the rotation point. In that case the equivalent applied torque can be estimated from equation (2.2). Note that the application point of the force should be far enough from the rotation axis such that the resulting bending motion is much larger than the displacement due to shear. Such effect is studied in Section 2.3.

$$T_y = hF_x, \quad T_x = hF_y \quad (2.2)$$

Similarly, instead of directly measuring the bending motion  $\theta_y$  of the flexible joint, its linear motion  $d_x$  at a certain height  $h$  from the rotation points is measured. The equivalent rotation is estimated from (2.3).

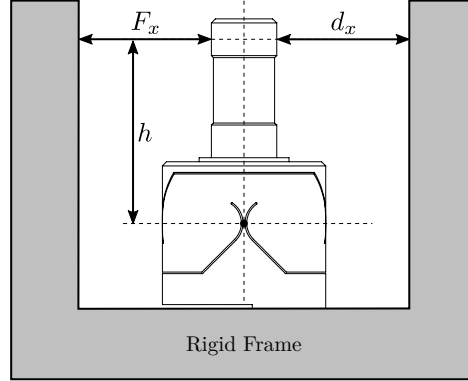
$$\theta_y = \tan^{-1} \left( \frac{d_x}{h} \right) \approx \frac{d_x}{h}, \quad \theta_x = \tan^{-1} \left( \frac{d_y}{h} \right) \approx \frac{d_y}{h} \quad (2.3)$$

Then, the bending stiffness can be estimated from (2.4).

$$k_{R_x} = \frac{T_x}{\theta_x} = \frac{hF_y}{\tan^{-1} \left( \frac{d_y}{h} \right)} \approx h^2 \frac{F_y}{d_y} \quad (2.4a)$$

$$k_{R_y} = \frac{T_y}{\theta_y} = \frac{hF_x}{\tan^{-1} \left( \frac{d_x}{h} \right)} \approx h^2 \frac{F_x}{d_x} \quad (2.4b)$$

The working principle of the measurement bench is schematically shown in Figure 2.1. One part of the flexible joint is fixed to a rigid frame while a (known) force  $F_x$  is applied to the other side of the flexible joint. The deflection of the joint  $d_x$  is measured by a displacement sensor.



**Figure 2.1:** Working principle of the test bench used to estimate the bending stiffness  $k_{R_y}$  of the flexible joints by measuring  $F_x$ ,  $d_x$  and  $h$

**Required external applied force** The bending stiffness is foreseen to be  $k_{R_y} \approx k_{R_x} \approx 5 \frac{Nm}{rad}$  and its stroke  $\theta_{y,max} \approx \theta_{x,max} \approx 25 mrad$ . The height between the flexible point (center of the joint) and the point where external forces are applied is  $h = 22.5 mm$  (see Figure 2.1).

The bending  $\theta_y$  of the flexible joint due to the force  $F_x$  is given by equation (2.5).

$$\theta_y = \frac{T_y}{k_{R_y}} = \frac{F_x h}{k_{R_y}} \quad (2.5)$$

Therefore, the force that has to be applied to test the full range of the flexible joint is given by equation (2.6). The measurement range of the force sensor should then be higher than 5.5 N.

$$F_{x,max} = \frac{k_{R_y} \theta_{y,max}}{h} \approx 5.5 N \quad (2.6)$$

**Required actuator stroke and sensors range** The flexible joint is designed to allow a bending motion of  $\pm 25 mrad$ . The corresponding stroke at the location of the force sensor is given by (2.7). In order to test the full range of the flexible joint, the means of applying a force (explained in the next section) should allow a motion of at least 0.5 mm. Similarly, the measurement range of the displacement sensor should also be higher than 0.5 mm.

$$d_{x,max} = h \tan(R_{x,max}) \approx 0.5 mm \quad (2.7)$$



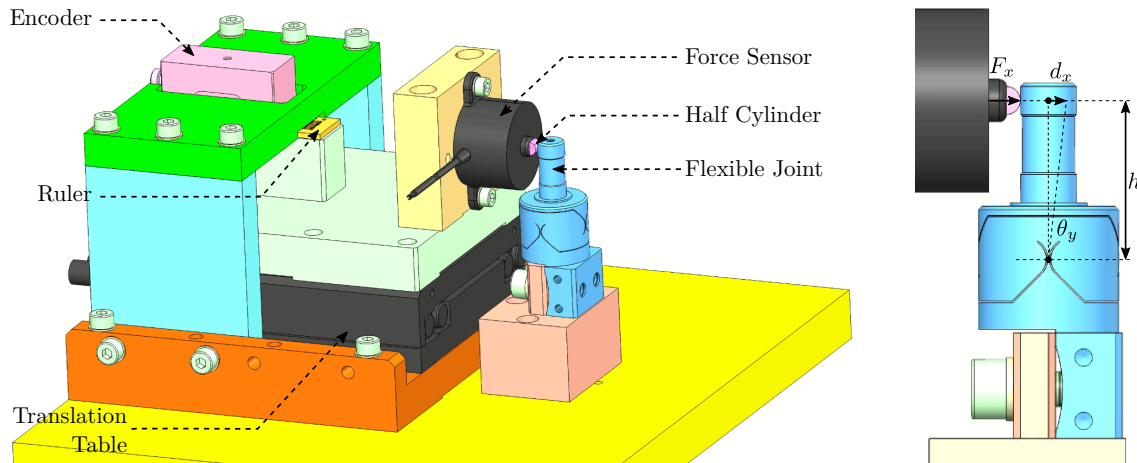
## 2.2 Developed test bench

As explained in Section 2.1, the flexible joint's bending stiffness is estimated by applying a known force to the flexible joint's tip and by measuring its deflection at the same point.

The force is applied using a load cell<sup>1</sup> such that the applied force to the flexible joint's tip is directly measured. In order to control the height and direction of the applied force, a cylinder cut in half is fixed at the tip of the force sensor (pink element in Figure 2.2b) that initially had a flat surface. This way, the contact between the flexible joint cylindrical tip and the force sensor is a point (intersection of two cylinders) at a precise height, and the force is applied in a known direction. To translate the load cell at a constant height, it is fixed to a translation stage<sup>2</sup> which is moved by hand.

Instead of measuring the displacement directly at the tip of the flexible joint (with a probe or an interferometer for instance), the displacement of the load cell itself is measured. To do so, an encoder<sup>3</sup> is used that measures the motion of a ruler. This ruler is fixed to the translation stage in line (i.e. at the same height) with the application point to reduce Abbe errors (see Figure 2.2a). Indirectly measuring the deflection of the flexible joint induces some errors due to the limited stiffness between the tip of the force sensor and the ruler. Such error will be estimated in Section 2.3.

The flexible joint can be rotated by 90 degrees in order to measure the bending stiffness in the two directions. The obtained CAD design of the measurement bench is shown in Figure 2.2a while a zoom on the flexible joint with the associated important quantities is shown in Figure 2.2b.



(a) Schematic of the test bench to measure the bending stiffness of the flexible joints (b) Zoom

**Figure 2.2:** CAD view of the test bench developed to measure the bending stiffness of the flexible joints. Different parts are shown in (a) while a zoom on the flexible joint is shown in (b)

<sup>1</sup>The load cell is FC22 from TE Connectivity. Measurement range is 50 N. Specified accuracy is 1% of the full range

<sup>2</sup>V-408 PIMag<sup>®</sup> linear stage is used. Crossed roller are used to guide the motion.

<sup>3</sup>Resolute<sup>™</sup> encoder with 1 nm resolution and  $\pm 40$  nm maximum non-linearity

## 2.3 Error budget

In order to estimate the accuracy of the measured bending stiffness that can be obtained using this measurement bench, an error budget is performed.

Based on equation (2.4), several errors can impact the accuracy of the measured bending stiffness:

- Errors in the measured torque  $M_x, M_y$ : this is mainly due to inaccuracies of the load cell and of the height estimation  $h$
- Errors in the measured bending motion of the flexible joints  $\theta_x, \theta_y$ : errors from limited shear stiffness, from the deflection of the load cell itself, and from inaccuracy of the height estimation  $h$

Let's first estimate the displacement induced only by the bending stiffness.

$$d_{x,b} = h \tan(\theta_y) = h \tan\left(\frac{F_x \cdot h}{k_{R_y}}\right) \quad (2.8)$$

**Effect of Shear** The applied force  $F_x$  will induce some shear  $d_{x,s}$  which is described by (2.9) with  $k_s$  the shear stiffness of the flexible joint.

$$d_{x,s} = \frac{F_x}{k_s} \quad (2.9)$$

The measured displacement  $d_x$  is affected by the shear as shown in equation (2.10).

$$d_x = d_{x,b} + d_{x,s} = h \tan\left(\frac{F_x \cdot h}{k_{R_y}}\right) + \frac{F_x}{k_s} \approx F_x \left(\frac{h^2}{k_{R_y}} + \frac{1}{k_s}\right) \quad (2.10)$$

The estimated bending stiffness  $k_{\text{est}}$  then depends on the shear stiffness (2.11).

$$k_{R_y, \text{est}} = h^2 \frac{F_x}{d_x} \approx k_{R_y} \frac{1}{1 + \frac{k_{R_y}}{k_s h^2}} \approx k_{R_y} \underbrace{\left(1 - \frac{k_{R_y}}{k_s h^2}\right)}_{\epsilon_s} \quad (2.11)$$

With an estimated shear stiffness  $k_s = 13 \text{ N}/\mu\text{m}$  from the finite element model and an height  $h = 25 \text{ mm}$ , the estimation errors of the bending stiffness due to shear is  $\epsilon_s < 0.1 \%$

**Effect of load cell limited stiffness** As explained in the previous section, because the measurement of the flexible joint deflection is indirectly made with the encoder, errors will be made if the load cell experiences some compression.

Suppose the load cell has an internal stiffness  $k_f$ , the same reasoning that was made for the effect of shear can be applied here. The estimation error of the bending stiffness due to the limited stiffness of the load cell is then described by (2.12).

$$k_{R_y, \text{est}} = h^2 \frac{F_x}{d_x} \approx k_{R_y} \frac{1}{1 + \frac{k_{R_y}}{k_F h^2}} \approx k_{R_y} \left( 1 - \underbrace{\frac{k_{R_y}}{k_F h^2}}_{\epsilon_f} \right) \quad (2.12)$$

With an estimated load cell stiffness of  $k_f \approx 1 \text{ N}/\mu\text{m}$  (from the documentation), the errors due to the load cell limited stiffness is around  $\epsilon_f = 1 \%$ .

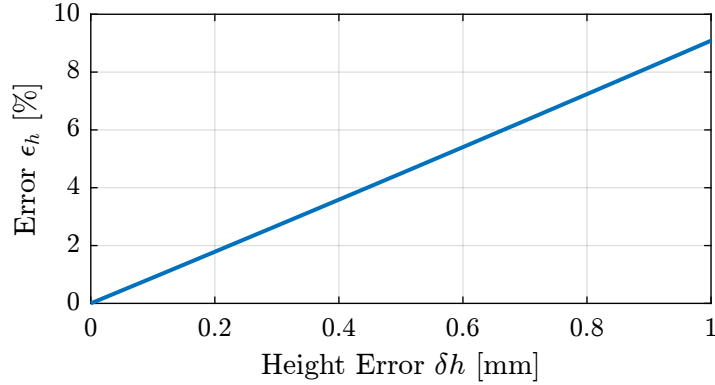
**Estimation error due to height estimation error** Let's consider an error  $\delta h$  in the estimation of the height  $h$  as described by (2.13).

$$h_{\text{est}} = h + \delta h \quad (2.13)$$

The computed bending stiffness will be (2.14).

$$k_{R_y, \text{est}} \approx h_{\text{est}}^2 \frac{F_x}{d_x} \approx k_{R_y} \left( 1 + 2 \underbrace{\frac{\delta h}{h} + \frac{\delta h^2}{h^2}}_{\epsilon_h} \right) \quad (2.14)$$

The bending stiffness error  $\epsilon_h$  due to the height estimation error  $\delta h$  is shown in Figure 2.3. The height estimation is foreseen to be reasonably accurate to within  $\pm 0.4 \text{ mm}$  which should make the bending stiffness error  $\epsilon_h < 3.5 \%$ .



**Figure 2.3:** Measured stiffness error due to height estimation error

**Estimation error due to force and displacement sensors accuracy** The maximum error of the measured displacement due the encoder non-linearity is  $40 \text{ nm}$ . As the measured displacement is foreseen to be  $0.5 \text{ mm}$ , the error  $\epsilon_d$  due to the encoder non linearity is very small  $\epsilon_d < 0.01 \%$ .

The accuracy of the load cell is specified at  $1 \%$  and therefore, estimation errors of the bending stiffness due to the limited load cell accuracy should be  $\epsilon_F < 1 \%$

**Conclusion** The different sources of errors are summarized in Table 2.1. The most important source of error comes from estimation error of the distance between the flexible joint rotation axis and its contact with the force sensor. An overall accuracy of  $\approx 5\%$  can be expected with this measurement bench, which should be enough for a first estimation of the bending stiffness of the flexible joints.

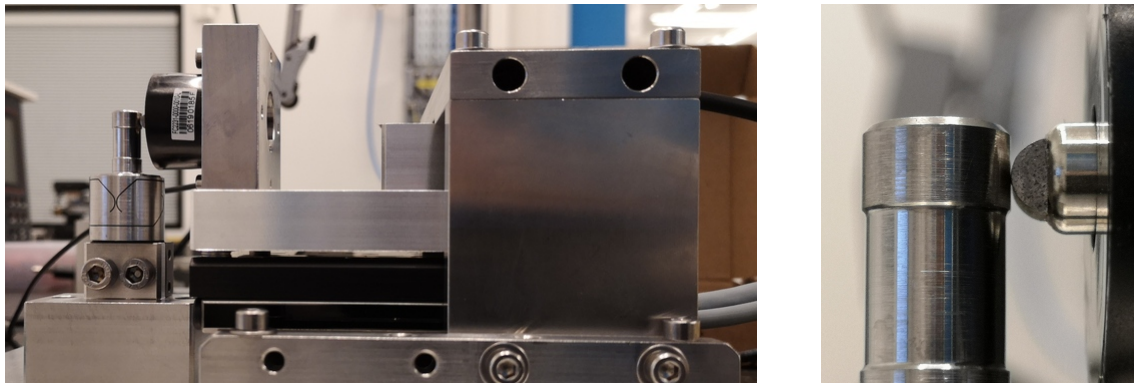
<b>Effect</b>	<b>Error</b>
Shear effect	$\epsilon_s < 0.1\%$
Load cell compliance	$\epsilon_f = 1\%$
Height error	$\epsilon_h < 3.5\%$
Displacement sensor	$\epsilon_d < 0.01\%$
Force sensor	$\epsilon_F < 1\%$

**Table 2.1:** Summary of the error budget for the estimation of the bending stiffness

# 3 Bending Stiffness Measurement

## 3.1 Introduction

A picture of the bench used to measure the X-bending stiffness of the flexible joints is shown in Figure 3.1a. A closer view on the force sensor tip is shown in Figure 3.1b.



(a) Picture of the measurement bench

(b) Zoom on the tip

**Figure 3.1:** Caption with reference to sub figure (a)

## 3.2 Force Sensor Calibration

In order to estimate the measured errors of the load cell “FC2231”, it is compared against another load cell<sup>1</sup>.

There are both specified to have  $\pm 1\%$  of non-linearity over the full range.

The XFL212R has a spherical interface while the FC2231 has a flat surface. Therefore, we should have a nice point contact when using the two force sensors as shown in Figure 3.2.

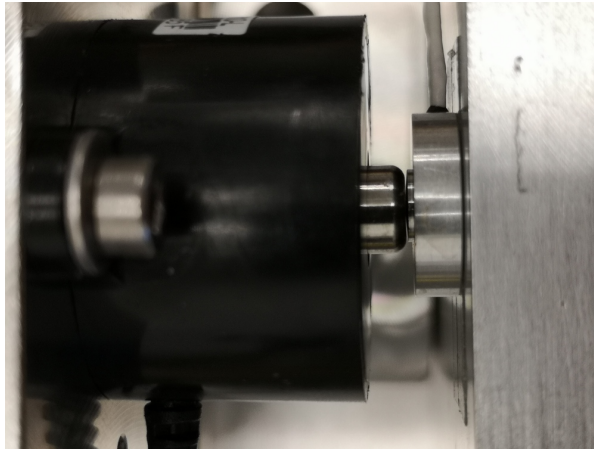
The two force sensors are therefore measuring the exact same force, and we can compare the two measurements.

Let’s select only the first part from the moment they are in contact until the maximum force is reached.

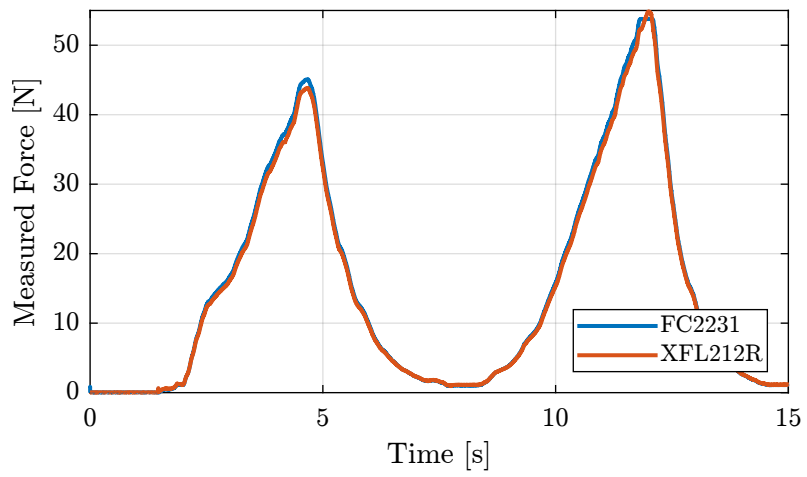
Then, let’s make a linear fit between the two measured forces.

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<sup>1</sup>XFL212R-50N from TE Connectivity. Measurement range is 50 N. Specified accuracy is 1% of the full range

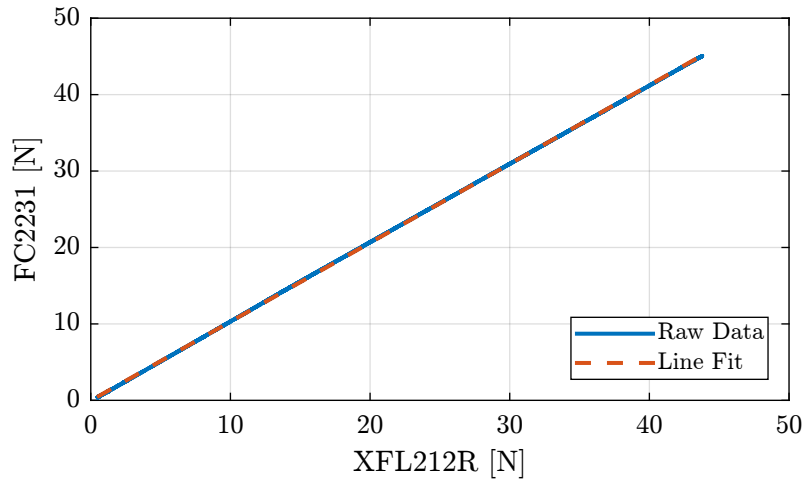


**Figure 3.2:** Zoom on the two force sensors in contact



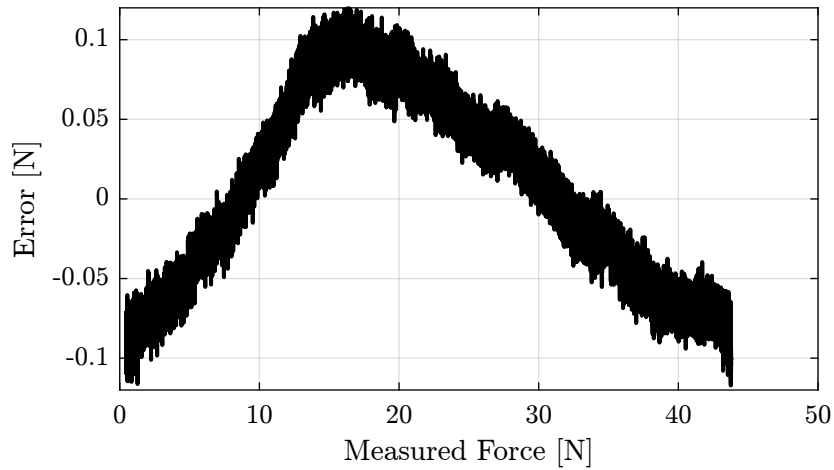
**Figure 3.3:** Measured force using both sensors as a function of time

The two forces are plotted against each other as well as the linear fit in Figure 3.4.



**Figure 3.4:** Measured two forces and linear fit

The measurement error between the two sensors is shown in Figure 3.5. It is below 0.1N for the full measurement range.



**Figure 3.5:** Error in Newtons

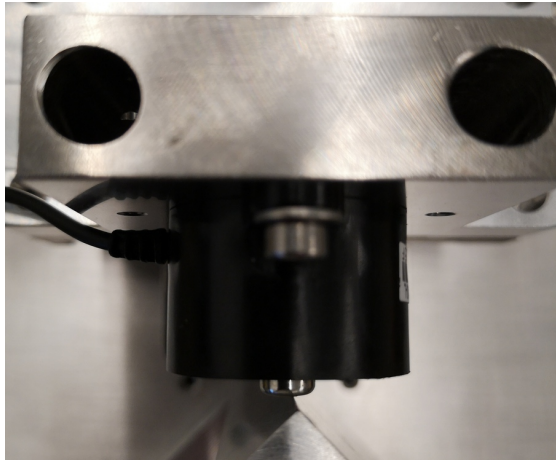
### 3.3 Force Sensor Stiffness

The objective of this measurement is to estimate the stiffness of the force sensor.

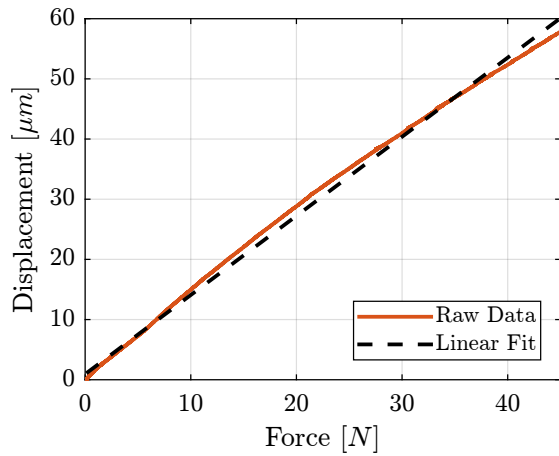
To do so, a very stiff element is fixed in front of the force sensor as shown in Figure 3.6a.

Then, we apply a force on the stiff element through the force sensor. We measure the deflection of the force sensor using an encoder.

Then, having the force and the deflection, we should be able to estimate the stiffness of the force sensor supposing the stiffness of the other elements are much larger.



(a) sub caption a



(b) sub caption b

**Figure 3.6:** Caption with reference to sub figure (a)

From the documentation, the deflection of the sensor at the maximum load (50N) is 0.05mm, the stiffness is therefore foreseen to be around  $1\text{ N}/\mu\text{m}$ .

The displacement as a function of the force as well as the linear fit are shown in Figure 3.6b.  $k_F \approx 0.75\text{ N}/\mu\text{m}$

### 3.4 Analysis of one measurement

The actual stiffness measurement is now performed by manually moving the translation stage from a start position where the force sensor is not yet in contact with the flexible joint to a position where flexible joint is on its mechanical stop.

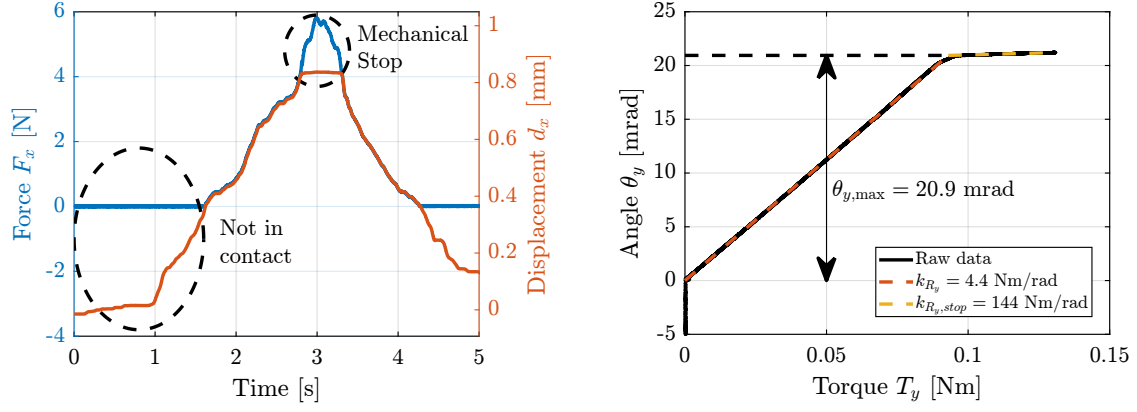
The measured force and displacement as a function of time are shown in Figure ??.

The angular motion  $\theta_y$  computed from the displacement  $d_x$  is displacement as function of the measured torque  $T_y$  in Figure ??. The bending stiffness can be estimated by computing the slope of the curve

The obtained characteristics are:

- Bending Stiffness:  $4.4\text{ Nm}/\text{rad}$
- Bending Stiffness at stop:  $144\text{ Nm}/\text{rad}$
- Bending Stroke:  $20.9\text{ mrad}$





(a) Force and displacement measured as a function of time (b) Angular displacement measured as a function of the applied torque

Figure 3.7: Caption with reference to sub figure (a)

### 3.5 Bending stiffness and bending stroke of all the flexible joints

Now, let's estimate the bending stiffness and stroke for all the flexible joints.

The results are summarized in Table 3.1 for the X direction and in Table 3.2 for the Y direction.

	$R_{R_x}$ [Nm/rad]	$k_{R_x,s}$ [Nm/rad]	$R_{x,max}$ [mrad]
1	5.5	173.6	18.9
2	6.1	195.0	17.6
3	6.1	191.3	17.7
4	5.8	136.7	18.3
5	5.7	88.9	22.0
6	5.7	183.9	18.7
7	5.7	157.9	17.9
8	5.8	166.1	17.9
9	5.8	159.5	18.2
10	6.0	143.6	18.1
11	5.0	163.8	17.7
12	6.1	111.9	17.0
13	6.0	142.0	17.4
14	5.8	130.1	17.9
15	5.7	170.7	18.6
16	6.0	148.7	17.5

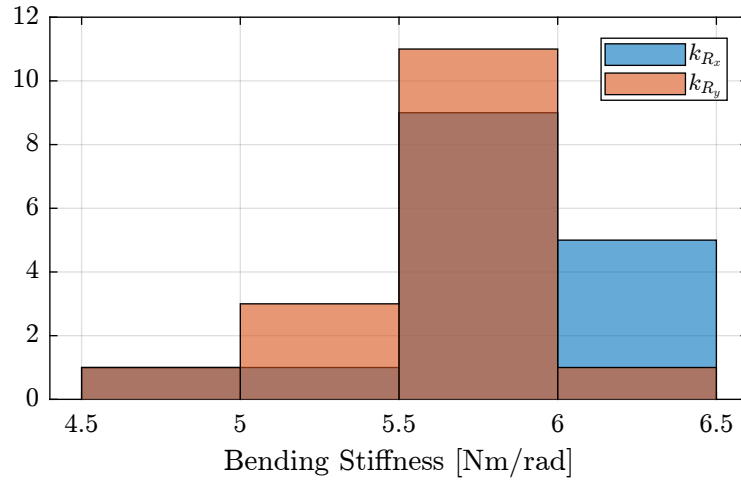
Table 3.1: Measured characteristics of the flexible joints in the X direction

### 3.6 Analysis

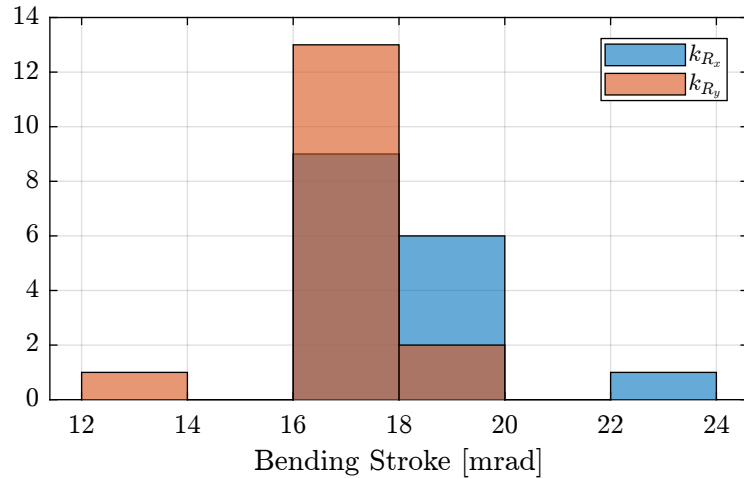
The dispersion of the measured bending stiffness is shown in Figure 3.8 and of the bending stroke in Figure 3.9.

	$R_{R_y}$ [Nm/rad]	$k_{R_y,s}$ [Nm/rad]	$R_{y,max}$ [mrad]
1	5.7	323.5	17.9
2	5.9	306.0	17.2
3	6.0	224.4	16.8
4	5.7	247.3	17.8
5	5.8	250.9	13.0
6	5.8	244.5	17.8
7	5.3	214.8	18.1
8	5.8	217.2	17.6
9	5.7	225.0	17.6
10	6.0	254.7	17.3
11	4.9	261.1	18.4
12	5.9	161.5	16.7
13	6.1	227.6	16.8
14	5.9	221.3	17.8
15	5.4	241.5	17.8
16	5.3	291.1	17.7

**Table 3.2:** Measured characteristics of the flexible joints in the Y direction

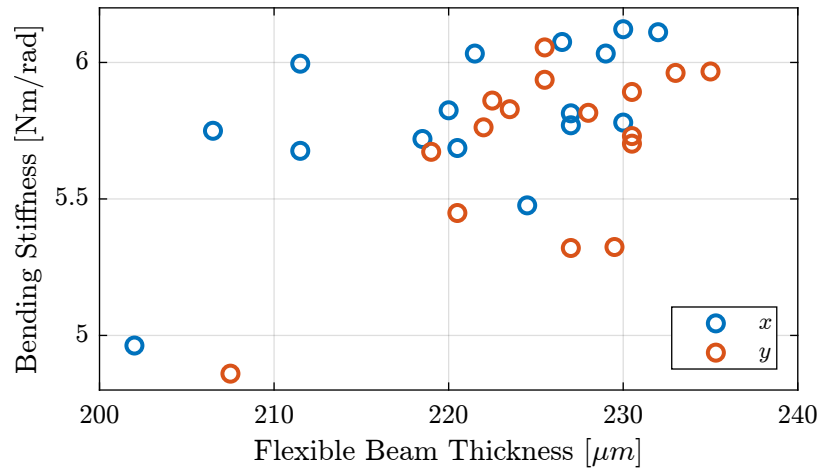


**Figure 3.8:** Histogram of the measured bending stiffness



**Figure 3.9:** Histogram of the measured bending stroke

The relation between the measured beam thickness and the measured bending stiffness is shown in Figure 3.10.



**Figure 3.10:** Measured bending stiffness as a function of the estimated flexible beam thickness

## Conclusion

### Important

The measured bending stiffness and bending stroke of the flexible joints are very close to the estimated one using a Finite Element Model.

The characteristics of all the flexible joints are also quite close to each other. This should allow us to model them with unique parameters.

## 4 Conclusion