

# **Flexible Joints - Test Bench**

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Ideally, these flexible joints would behave as perfect ball joints, that is to say:

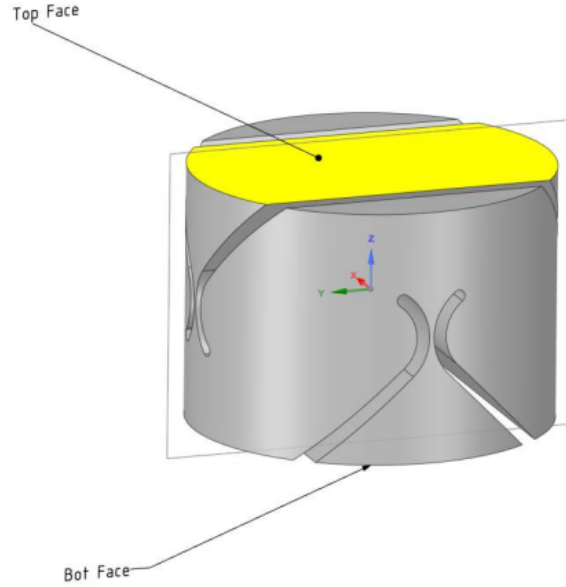
- no bending and torsional stiffnesses
- infinite shear and axial stiffnesses
- un-limited bending and torsional stroke
- no friction, no backlash

The real characteristics of the flexible joints will influence the dynamics of the Nano-Hexapod. Using a multi-body dynamical model of the nano-hexapod, the specifications in term of stiffness and stroke of the flexible joints have been determined and summarized in Table 2.

**Table 1:** Specifications for the flexible joints and estimated characteristics from the Finite Element Model

	Specification	FEM
Axial Stiffness	$> 100 N/\mu m$	94
Shear Stiffness	$> 1 N/\mu m$	13
Bending Stiffness	$< 100 Nm/rad$	5
Torsion Stiffness	$< 500 Nm/rad$	260
Bending Stroke	$> 1 mrad$	24.5
Torsion Stroke	$> 5 \mu rad$	

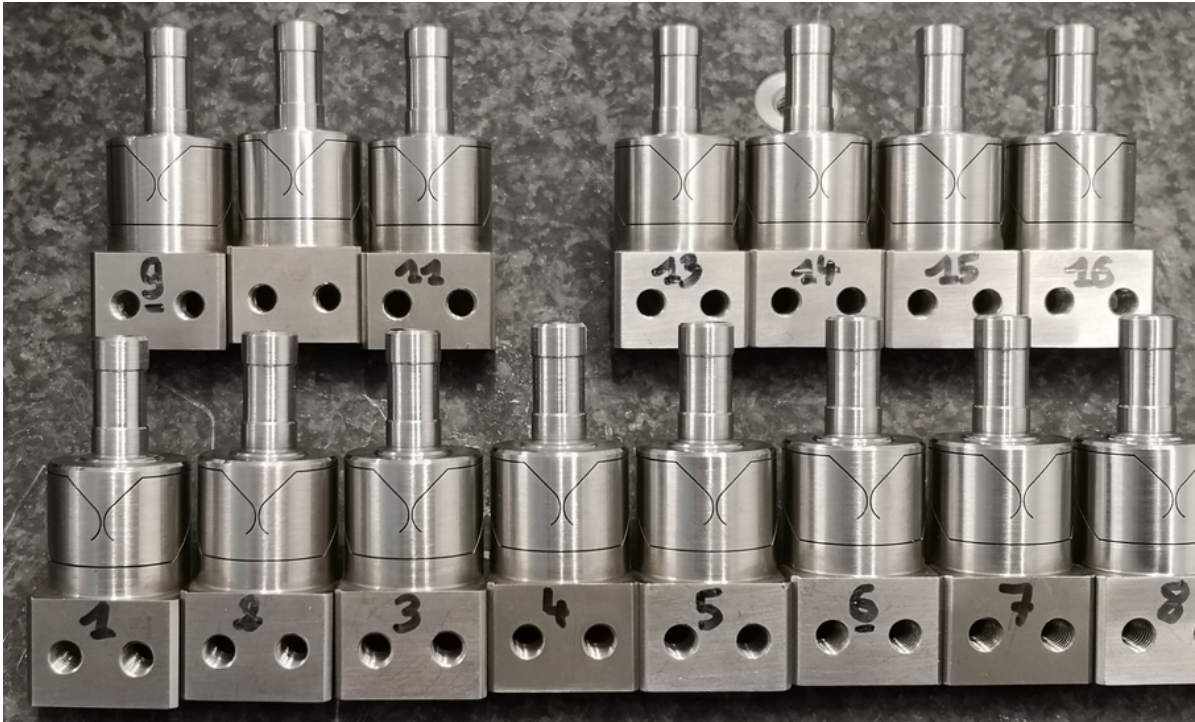
Then, the classical geometry of a flexible ball joint shown in Figure 1 has been optimized in order to meet the requirements. This has been done using a Finite Element Software and the obtained joint's characteristics are summarized in Table 2.



**Figure 1:** Flexible part of the Joint used for FEM - CAD view

The obtained geometry are defined in the [drawings of the flexible joints](#). The material is a special kind of stainless steel called "F16PH".

The flexible joints can be seen on Figure 2.



**Figure 2:** 15 of the 16 flexible joints

In this document, we present a test-bench that has been developed in order to measure the bending stiffness of flexible joints.

It is structured as follow:

- Section 1: each flexible joint is measured using a profile projector
- Section 2: the stiffness measurement bench is presented
- Section 3: an error budget is performed in order to estimate the accuracy of the measured stiffness
- Section 4: first measurements are performed
- Section 5: the bending stiffness of the flexible joints are measured

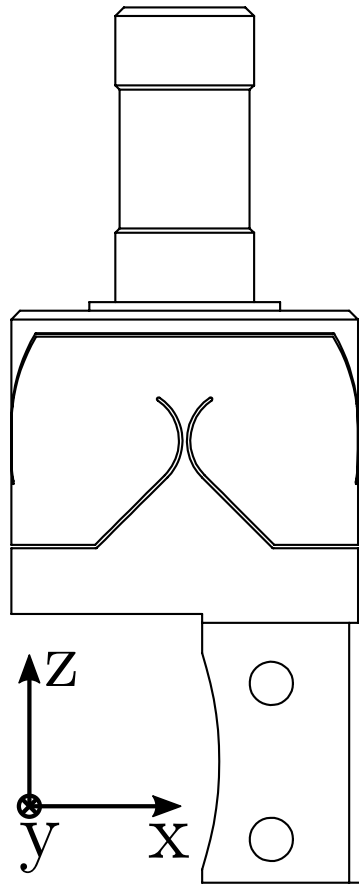
**Table 2:** Report sections and corresponding Matlab files

Sections	Matlab File
Section ??	test_joints_1.m

# 1 Dimensional Measurements

## 1.1 Measurement Bench

The axis corresponding to the flexible joints are defined in Figure 1.1.

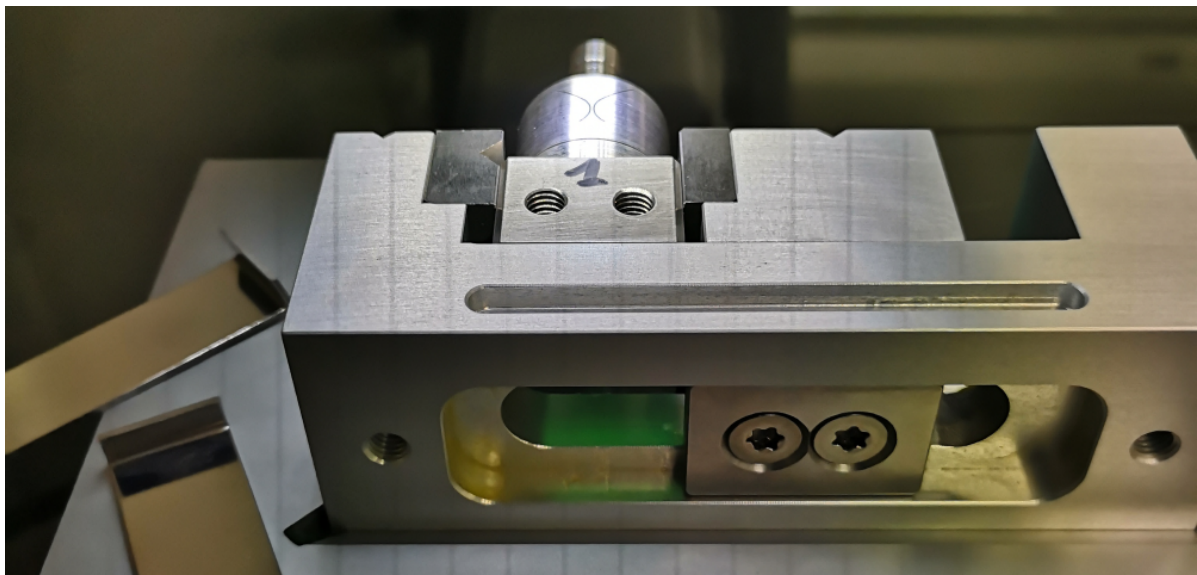


**Figure 1.1:** Define axis for the flexible joints

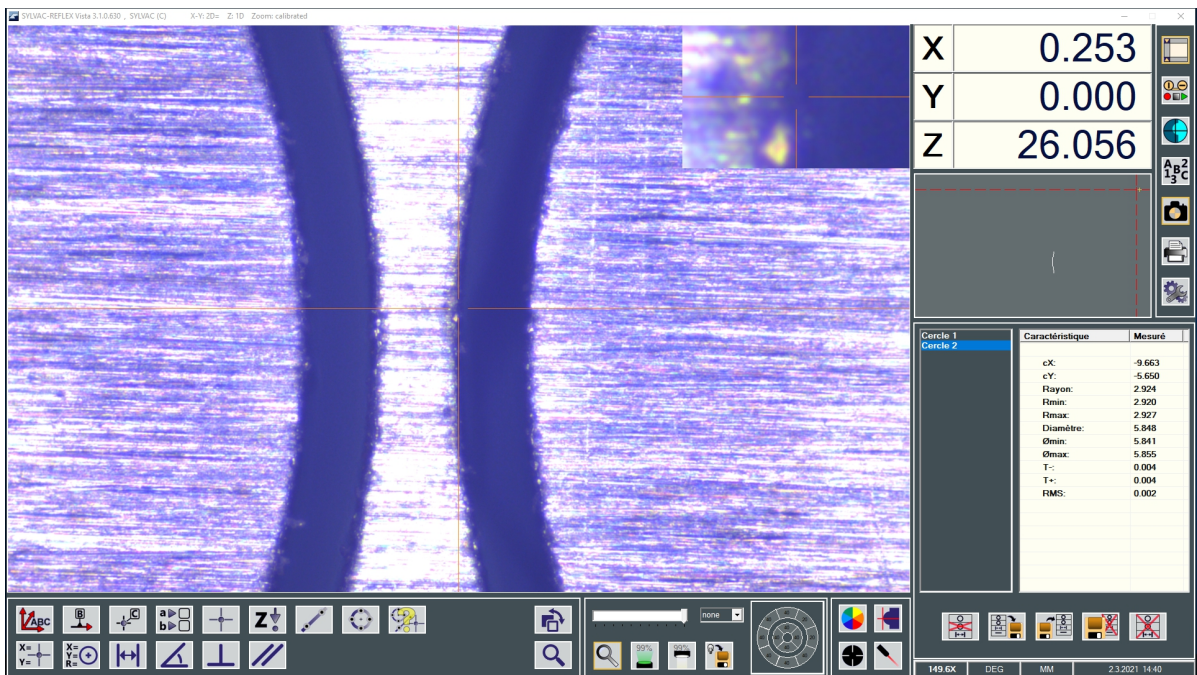
The dimensions of the flexible part in the Y-Z plane will contribute to the X-bending stiffness. Similarly, the dimensions of the flexible part in the X-Z plane will contribute to the Y-bending stiffness.

The setup to measure the dimension of the “X” flexible beam is shown in Figure 1.2.

What we typically observe is shown in Figure 1.3. It is then possible to estimate to dimension of the flexible beam with an accuracy of  $\approx 5 \mu m$ ,



**Figure 1.2:** Setup to measure the dimension of the flexible beam corresponding to the X-bending stiffness



**Figure 1.3:** Image used to measure the flexible joint's dimensions

## 1.2 Measurement Results

The expected flexible beam thickness is  $250 \mu m$ . However, it is more important that the thickness of all beams are close to each other.

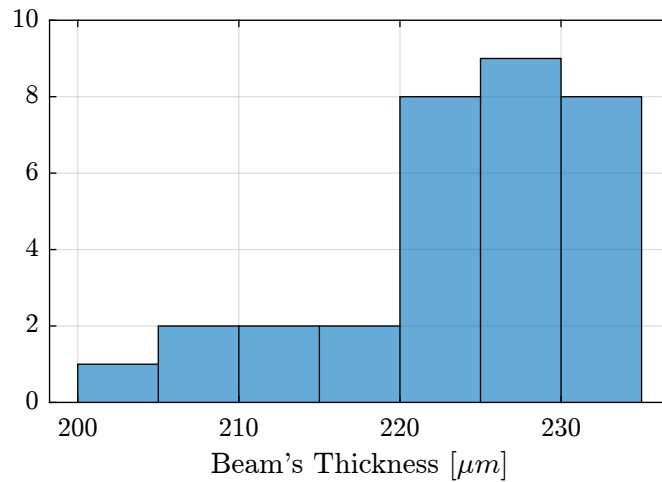
The dimension of the beams are been measured at each end to be able to estimate the mean of the beam thickness.

All the measured dimensions are summarized in Table 1.1.

**Table 1.1:** Measured Dimensions of the flexible beams in  $\mu m$

	Y1	Y2	X1	X2
1	223	226	224	214
2	229	231	237	224
3	234	230	239	231
4	233	227	229	232
5	225	212	228	228
6	220	221	224	220
7	206	207	228	226
8	230	224	224	223
9	223	231	228	233
10	228	230	235	231
11	197	207	211	204
12	227	226	225	226
13	215	228	231	220
14	216	224	224	221
15	209	214	220	221
16	213	210	230	229

An histogram of these measured dimensions is shown in Figure 1.4.



**Figure 1.4:** Histogram for the (16x2) measured beams' thickness

## 2 Measurement Test Bench - Bending Stiffness

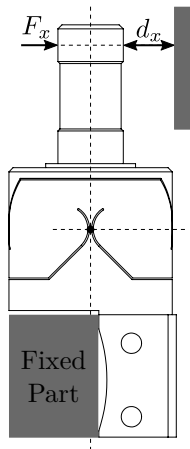
The most important characteristic of the flexible joint that we want to measure is its bending stiffness  $k_{R_x} \approx k_{R_y}$ .

To do so, we have to apply a torque  $T_x$  on the flexible joint and measure its angular deflection  $\theta_x$ . The stiffness is then

$$k_{R_x} = \frac{T_x}{\theta_x} \quad (2.1)$$

As it is quite difficult to apply a pure torque, a force will be applied instead. The application point of the force should be far enough from the flexible part such that the obtained bending is much larger than the displacement in shear.

The working principle of the bench is schematically shown in Figure 2.1. One part of the flexible joint is fixed. On the mobile part, a force  $F_x$  is applied which is equivalent to a torque applied on the flexible joint center. The induced rotation is measured with a displacement sensor  $d_x$ .



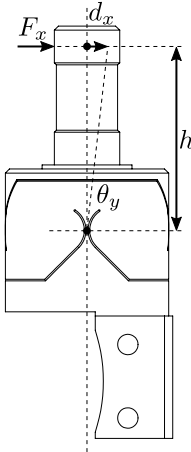
**Figure 2.1:** Test Bench - working principle

This test-bench will be used to have a first approximation of the bending stiffness and stroke of the flexible joints. Another test-bench, better engineered will be used to measure the flexible joint's characteristics with better accuracy.



## 2.1 Flexible joint Geometry

The flexible joint used for the Nano-Hexapod is shown in Figure 2.2. Its bending stiffness is foreseen to be  $k_{R_y} \approx 5 \frac{Nm}{rad}$  and its stroke  $\theta_{y,max} \approx 25 mrad$ .



**Figure 2.2:** Geometry of the flexible joint

The height between the flexible point (center of the joint) and the point where external forces are applied is  $h = 20 mm$ .

Let's define the parameters on Matlab.

## 2.2 Required external applied force

The bending  $\theta_y$  of the flexible joint due to the force  $F_x$  is:

$$\theta_y = \frac{M_y}{k_{R_y}} = \frac{F_x h}{k_{R_y}} \quad (2.2)$$

Therefore, the applied force to test the full range of the flexible joint is:

$$F_{x,max} = \frac{k_{R_y} \theta_{y,max}}{h} \quad (2.3)$$

And we obtain:

$$F_{x,max} = 6.2 [N] \quad (2.4)$$

The measurement range of the force sensor should then be higher than  $6.2 N$ .

## 2.3 Required actuator stroke and sensors range

The flexible joint is designed to allow a bending motion of  $\pm 25 \text{ mrad}$ . The corresponding stroke at the location of the force sensor is:

$$d_{x,\max} = h \tan(R_{x,\max})$$

$$d_{max} = 0.5 \text{ [mm]} \quad (2.5)$$

In order to test the full range of the flexible joint, the stroke of the translation stage used to move the force sensor should be higher than  $0.5 \text{ mm}$ . Similarly, the measurement range of the displacement sensor should also be higher than  $0.5 \text{ mm}$ .

## 2.4 Test Bench

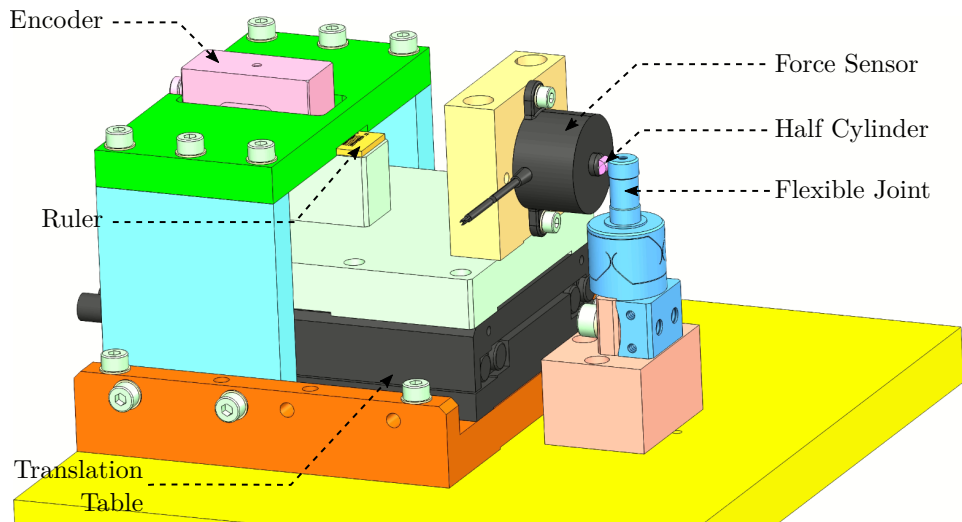
A CAD view of the measurement bench is shown in Figure 2.3.

### Note

Here are the different elements used in this bench:

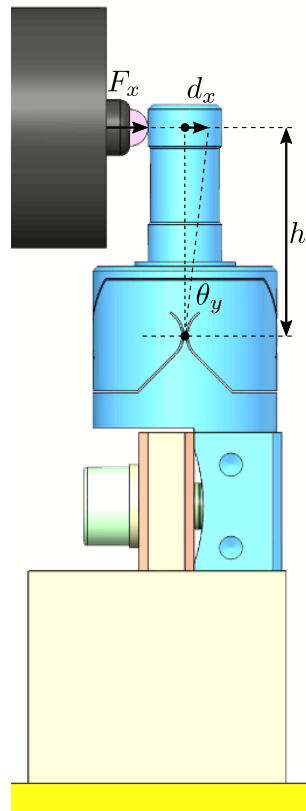
- **Translation Stage:** V-408
- **Load Cells:** FC2231-0000-0010-L
- **Encoder:** Renishaw Resolute 1nm

Both the measured force and displacement are acquired at the same time using a Speedgoat machine.



**Figure 2.3:** Schematic of the test bench to measure the bending stiffness of the flexible joints

A side view of the bench with the important quantities are shown in Figure 2.4.



**Figure 2.4:** Schematic of the test bench to measure the bending stiffness of the flexible joints

## 3 Error budget

Many things can impact the accuracy of the measured bending stiffness such as:

- Errors in the force and displacement measurement
- Shear effects
- Deflection of the Force sensor
- Errors in the geometry of the bench

In this section, we wish to estimate the attainable accuracy with the current bench, and identified the limiting factors.

### 3.1 Finite Element Model

From the Finite Element Model, the stiffness and stroke of the flexible joint have been computed and summarized in Tables 3.1 and 3.2.

**Table 3.1:** Axial/Shear characteristics

	Stiffness [N/um]	Max Force [N]	Stroke [um]
Axial	94	469	5
Shear	13	242	19

**Table 3.2:** Bending/Torsion characteristics

	Stiffness [Nm/rad]	Max Torque [Nmm]	Stroke [mrad]
Bending	5	118	24
Torsional	260	1508	6

### 3.2 Setup

The setup is schematically represented in Figure ??.

The force is applied on top of the flexible joint with a distance  $h$  with the joint's center. The displacement of the flexible joint is also measured at the same height.

The height between the joint's center and the force application point is:

### 3.3 Effect of Bending

The torque applied is:

$$M_y = F_x \cdot h \quad (3.1)$$

The flexible joint is experiencing a rotation  $\theta_y$  due to the torque  $M_y$ :

$$\theta_y = \frac{M_y}{k_{R_y}} = \frac{F_x \cdot h}{k_{R_y}} \quad (3.2)$$

This rotation is then measured by the displacement sensor. The measured displacement is:

$$D_b = h \tan(\theta_y) = h \tan\left(\frac{F_x \cdot h}{k_{R_y}}\right) \quad (3.3)$$

### 3.4 Computation of the bending stiffness

From equation (3.3), we can compute the bending stiffness:

$$k_{R_y} = \frac{F_x \cdot h}{\tan^{-1}\left(\frac{D_b}{h}\right)} \quad (3.4)$$

For small displacement, we have

$$\boxed{k_{R_y} \approx h^2 \frac{F_x}{d_x}} \quad (3.5)$$

And therefore, to precisely measure  $k_{R_y}$ , we need to:

- precisely measure the motion  $d_x$
- precisely measure the applied force  $F_x$
- precisely know the height of the force application point  $h$

### 3.5 Estimation error due to force and displacement sensors accuracy

The maximum error on the measured displacement with the encoder is 40 nm. This is quite negligible compared to the measurement range of 0.5 mm.

The accuracy of the force sensor is around 1% and therefore, we should expect to have an accuracy on the measured stiffness of at most 1%.

### 3.6 Estimation error due to Shear

The effect of Shear on the measured displacement is simply:

$$D_s = \frac{F_x}{k_s} \quad (3.6)$$

The measured displacement will be the effect of shear + effect of bending

$$d_x = D_b + D_s = h \tan\left(\frac{F_x \cdot h}{k_{R_y}}\right) + \frac{F_x}{k_s} \approx F_x \left(\frac{h^2}{k_{R_y}} + \frac{1}{k_s}\right) \quad (3.7)$$

The estimated bending stiffness  $k_{\text{est}}$  will then be:

$$k_{\text{est}} = h^2 \frac{F_x}{d_x} \approx k_{R_y} \frac{1}{1 + \frac{k_{R_y}}{k_s h^2}} \quad (3.8)$$

---

Results

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The measurement error due to Shear is 0.1 %

---

### 3.7 Estimation error due to force sensor compression

The measured displacement is not done directly at the joint's location. The force sensor compression will then induce an error on the joint's stiffness.

The force sensor stiffness  $k_F$  is estimated to be around:

---

Results

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$k_F = 1.0e+06$  [N/m]

---

The measured displacement will be the sum of the displacement induced by the bending and by the compression of the force sensor:

$$d_x = D_b + \frac{F_x}{k_F} = h \tan\left(\frac{F_x \cdot h}{k_{R_y}}\right) + \frac{F_x}{k_F} \approx F_x \left(\frac{h^2}{k_{R_y}} + \frac{1}{k_F}\right) \quad (3.9)$$

The estimated bending stiffness  $k_{\text{est}}$  will then be:

$$k_{\text{est}} = h^2 \frac{F_x}{d_x} \approx k_{R_y} \frac{1}{1 + \frac{k_{R_y}}{k_F h^2}} \quad (3.10)$$

---

Results

---

The measurement error due to height estimation errors is 0.8 %

---

### 3.8 Estimation error due to height estimation error

Let's consider an error in the estimation of the height from the application of the force to the joint's center:

$$h_{\text{est}} = h(1 + \epsilon) \quad (3.11)$$

The computed bending stiffness will be:

$$k_{\text{est}} \approx h_{\text{est}}^2 \frac{F_x}{d_x} \quad (3.12)$$

And the stiffness estimation error is:

$$\frac{k_{\text{est}}}{k_{R_y}} = (1 + \epsilon)^2 \quad (3.13)$$

---

Results

---

The measurement error due to height estimation errors of 0.2 [mm] is 1.6 %

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### 3.9 Conclusion

Based on the above analysis, we should expect no better than few percent of accuracy using the current test-bench. This is well enough for a first estimation of the bending stiffness of the flexible joints.

Another measurement bench allowing better accuracy will be developed.

## 4 First Measurements

- Encoder: [Renishaw Resolute 1mm](#)

### 4.1 Force Sensor Calibration

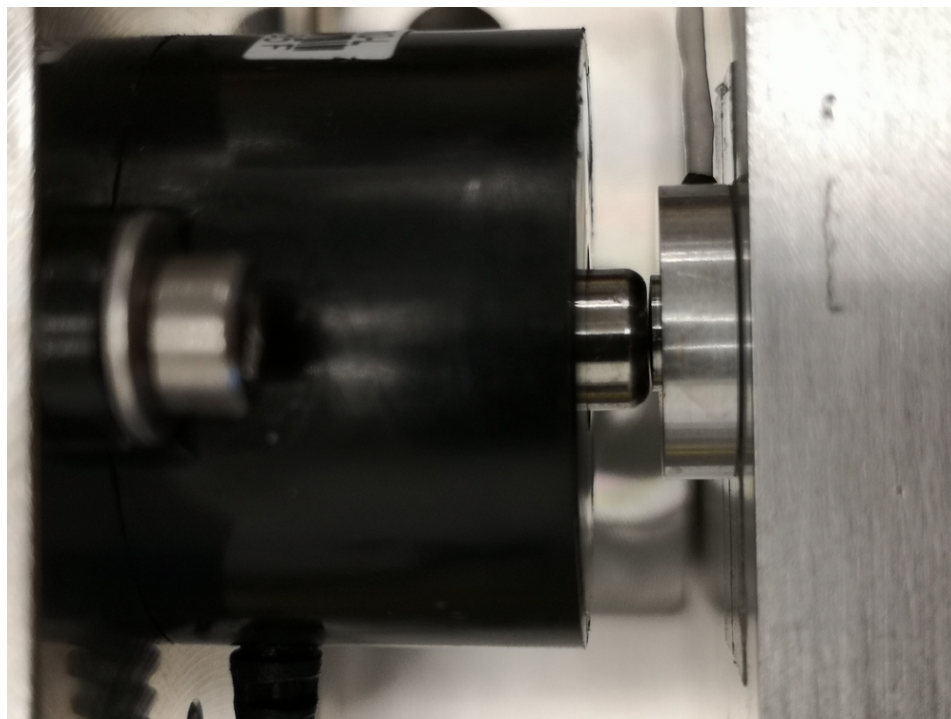
#### Note

##### Load Cells:

- [FC2231-0000-0010-L](#)
- [XFL212R](#)

There are both specified to have  $\pm 1\%$  of non-linearity over the full range.

The XFL212R has a spherical interface while the FC2231 has a flat surface. Therefore, we should have a nice point contact when using the two force sensors as shown in Figure 4.4.

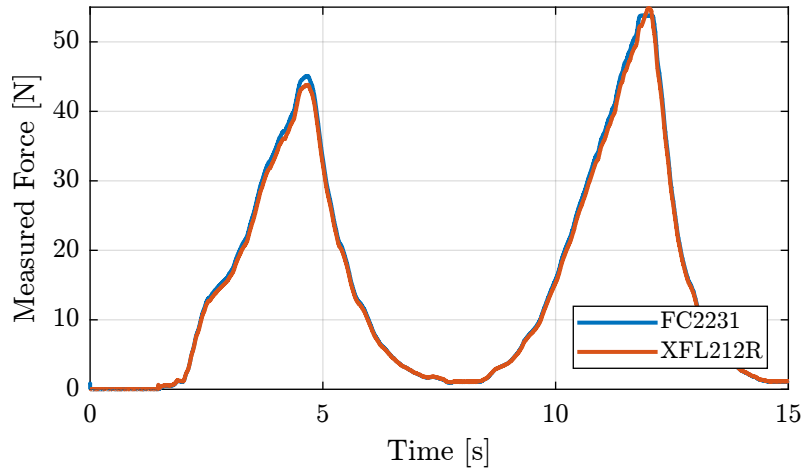


**Figure 4.1:** Zoom on the two force sensors in contact



The two force sensors are therefore measuring the exact same force, and we can compare the two measurements.

Let's load the measured force of both sensors. We remove any offset such that they are both measuring no force when not in contact.

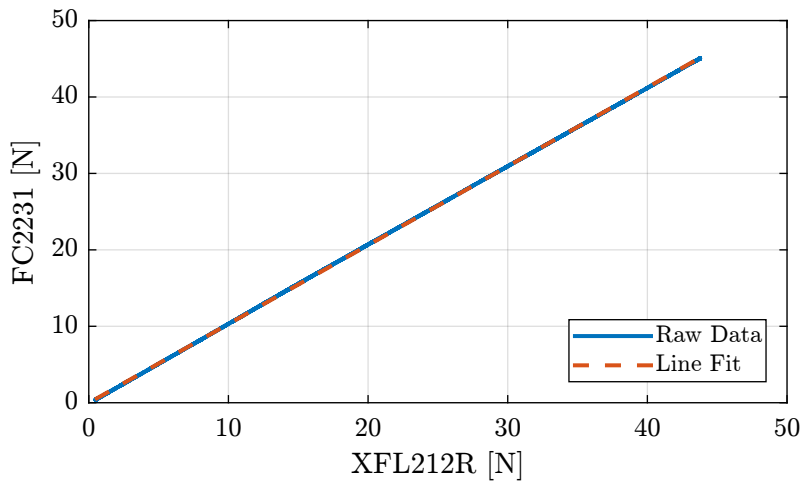


**Figure 4.2:** Measured force using both sensors as a function of time

Let's select only the first part from the moment they are in contact until the maximum force is reached.

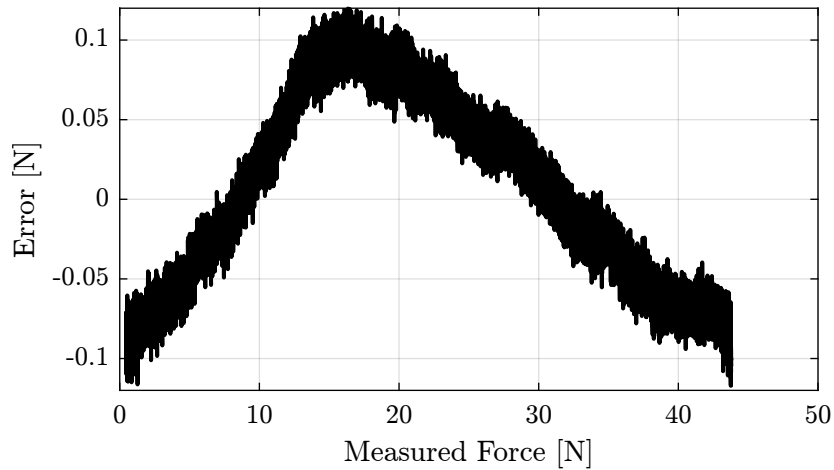
Then, let's make a linear fit between the two measured forces.

The two forces are plotted against each other as well as the linear fit in Figure 4.6.



**Figure 4.3:** Measured two forces and linear fit

The measurement error between the two sensors is shown in Figure 4.7. It is below 0.1N for the full measurement range.



**Figure 4.4:** Error in Newtons

## 4.2 Force Sensor Stiffness

The objective of this measurement is to estimate the stiffness of the force sensor [FC2231-0000-0010-L](#).

To do so, a very stiff element is fixed in front of the force sensor as shown in [Figure 4.8](#).

Then, we apply a force on the stiff element through the force sensor. We measure the deflection of the force sensor using an encoder.

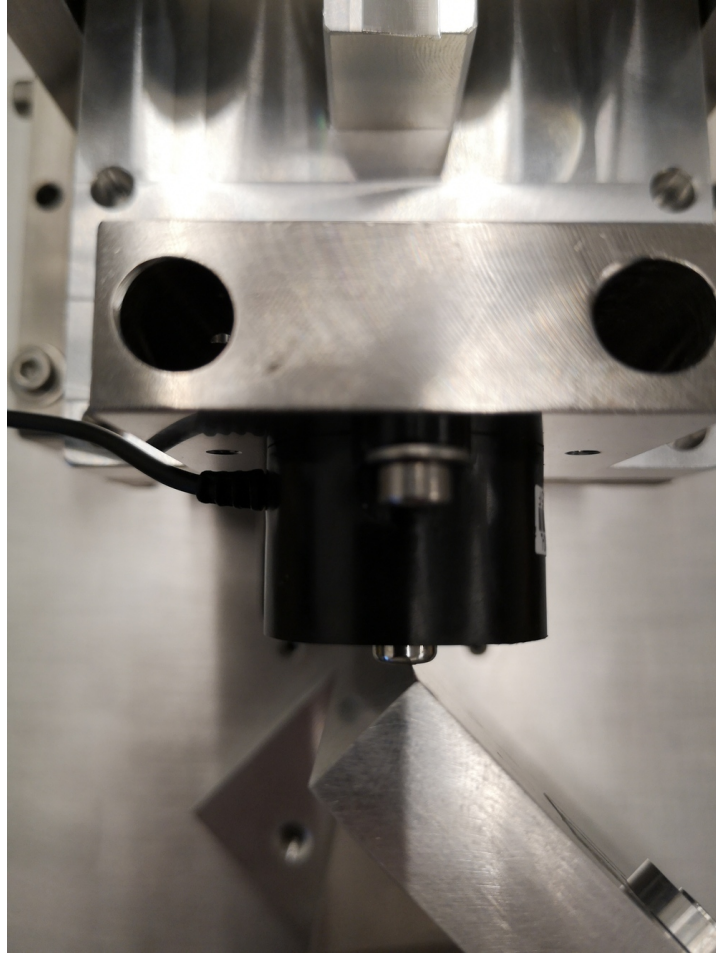
Then, having the force and the deflection, we should be able to estimate the stiffness of the force sensor supposing the stiffness of the other elements are much larger.

From the documentation, the deflection of the sensor at the maximum load (50N) is 0.05mm, the stiffness is therefore foreseen to be around  $1 \text{ N}/\mu\text{m}$ .

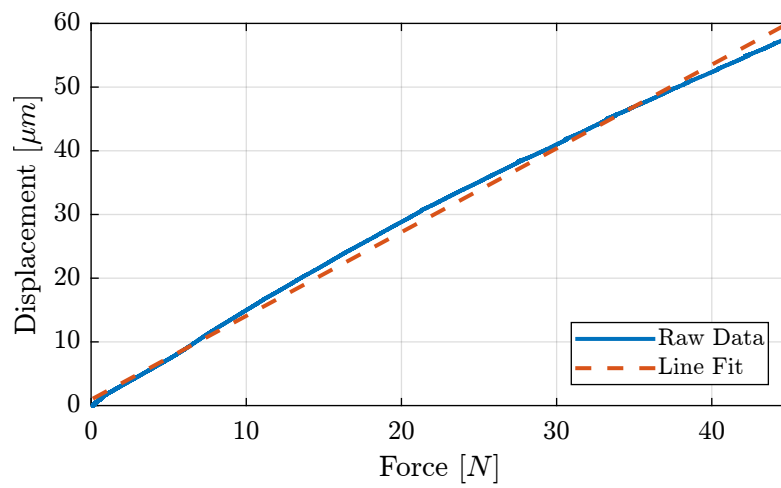
Let's load the measured force as well as the measured displacement. Some pre-processing is applied on the data. The linear fit is performed. The displacement as a function of the force as well as the linear fit are shown in [Figure 4.9](#).

And we obtain the following stiffness:

Results	
$k = 0.76 \text{ [N}/\mu\text{m]}$	



**Figure 4.5:** Bench used to measured the stiffness of the force sensor

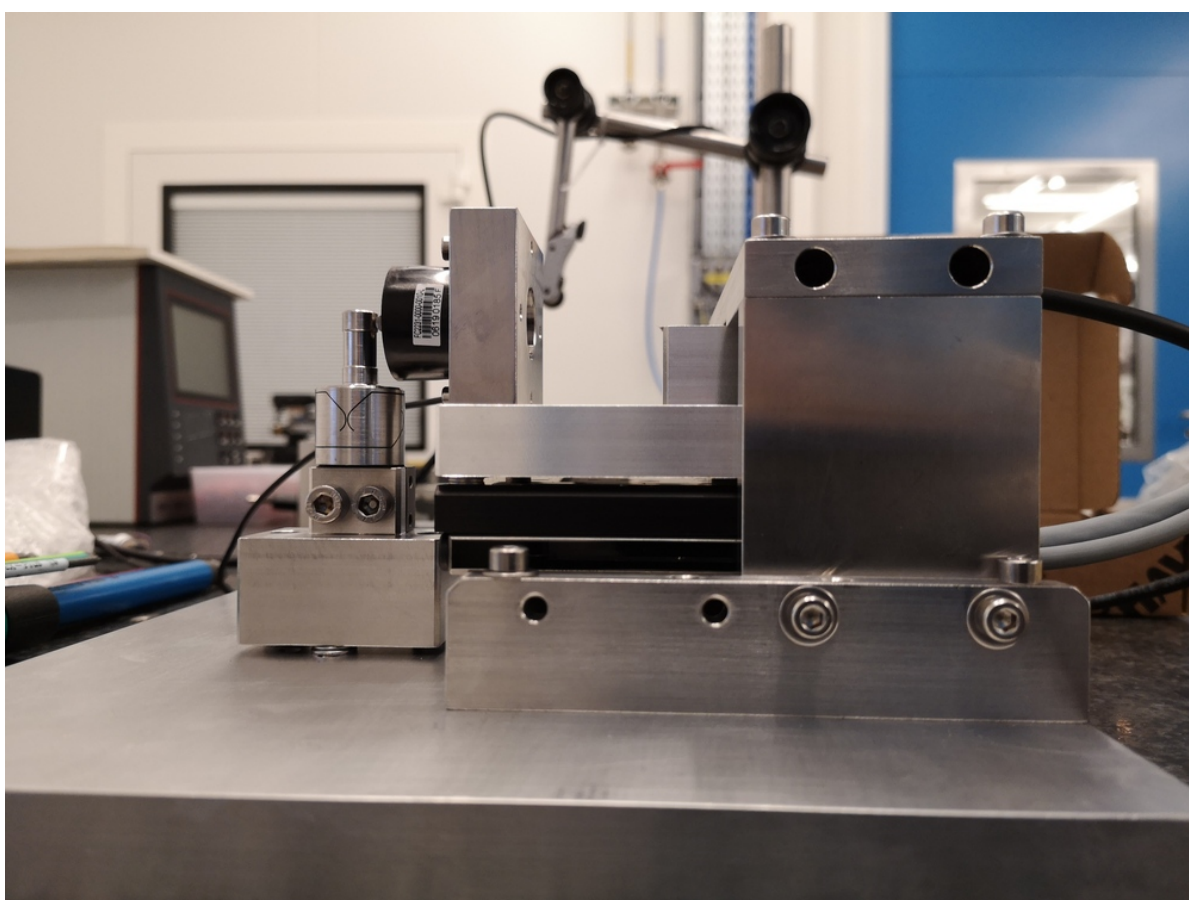


**Figure 4.6:** Displacement as a function of the measured force

# 5 Bending Stiffness Measurement

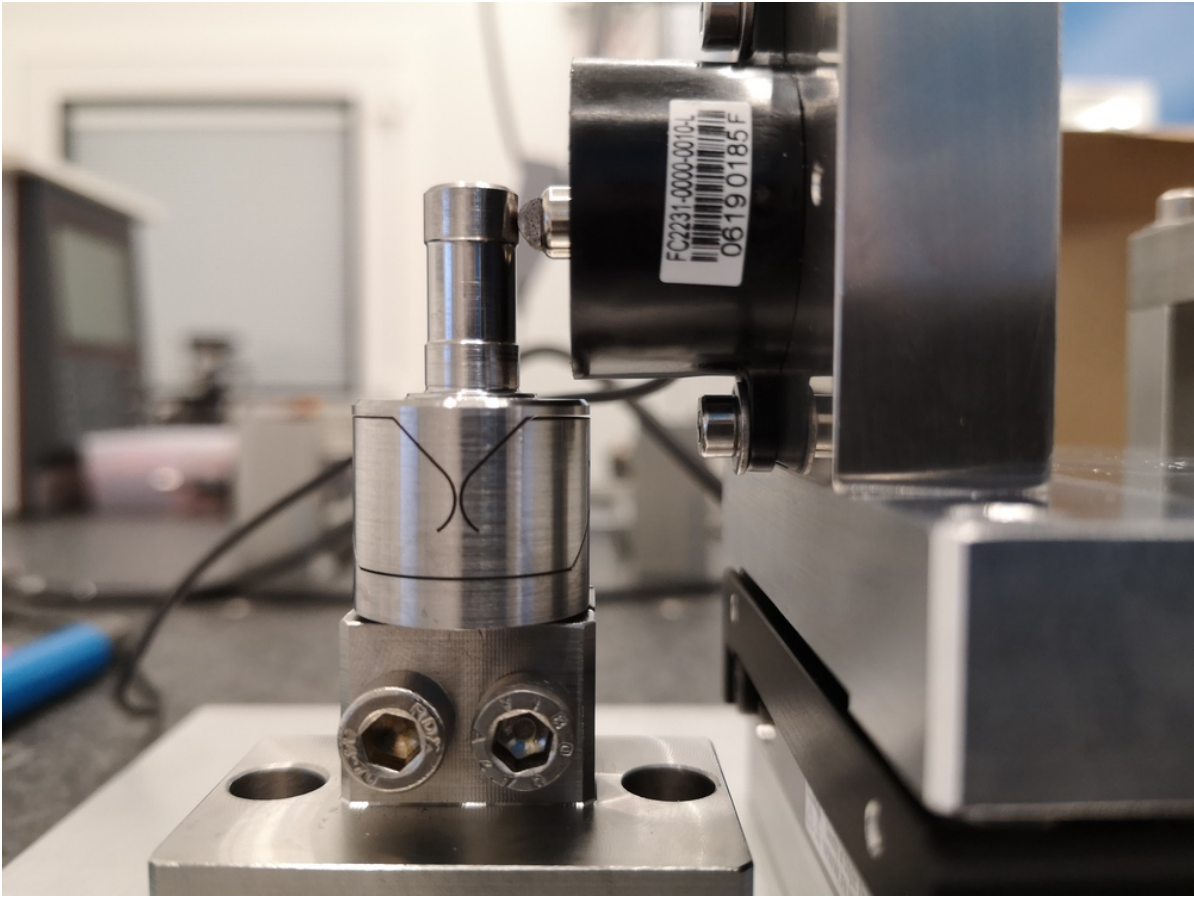
## 5.1 Introduction

A picture of the bench used to measure the X-bending stiffness of the flexible joints is shown in Figure 5.1. A closer view on flexible joint is shown in Figure 5.2 and a zoom on the force sensor tip is shown in Figure 5.3.

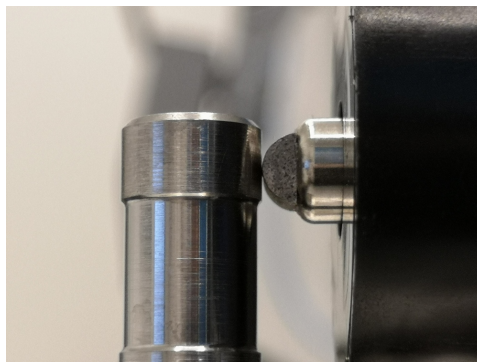


**Figure 5.1:** Side view of the flexible joint stiffness bench. X-Bending stiffness is measured.

The same bench used to measure the Y-bending stiffness of the flexible joint by pivoting the flexible joint by 90 degrees.



**Figure 5.2:** Zoom on the flexible joint - Side view



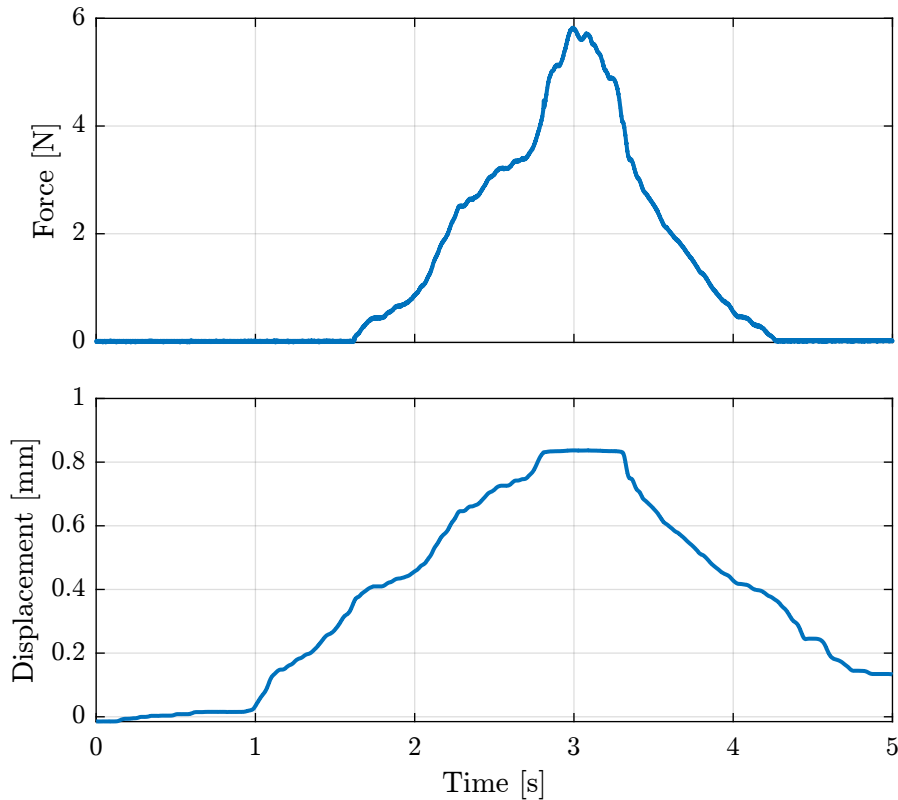
**Figure 5.3:** Zoom on the tip of the force sensor

## 5.2 Analysis of one measurement

In this section is shown how the data are analysis in order to measured:

- the bending stiffness
- the bending stroke
- the stiffness once the mechanical stops are in contact

The height from the flexible joint's center and the point of application force  $h$  is defined below: The obtained time domain measurements are shown in Figure 5.4.



**Figure 5.4:** Typical time domain measurements

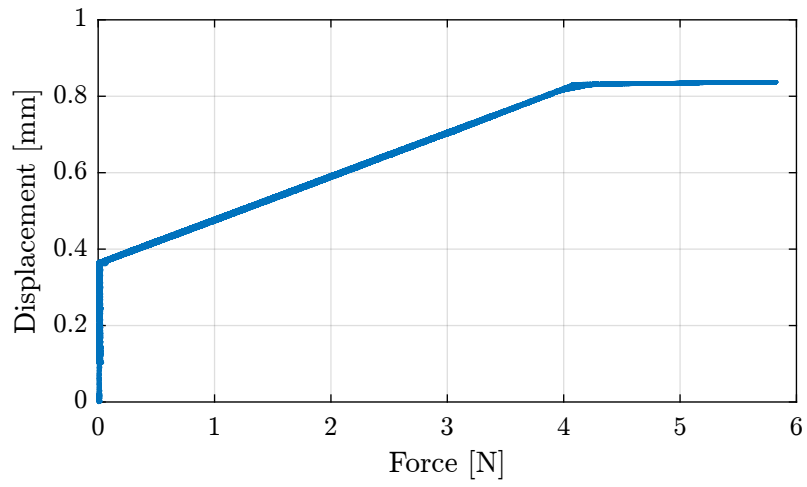
The displacement as a function of the force is then shown in Figure 5.5.

The bending stiffness can be estimated by computing the slope of the curve in Figure 5.5. The bending stroke and the stiffness when touching the mechanical stop can also be estimated from the same figure.

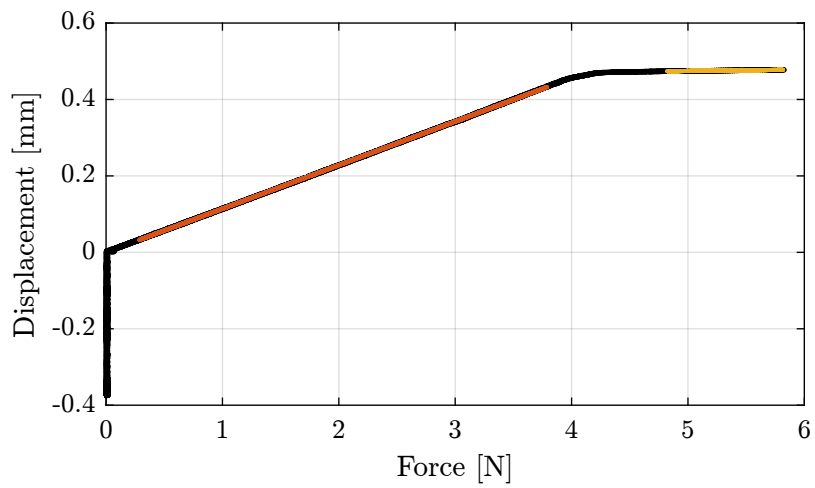
The raw data as well as the fit corresponding to the two stiffnesses are shown in Figure 5.6.

Then, the bending stroke is estimated as crossing point between the two fitted lines: The obtained characteristics are:

- Bending Stiffness: 5.5Nm/rad



**Figure 5.5:** Typical measurement of the displacement as a function of the applied force



**Figure 5.6:** Typical measurement of the displacement as a function of the applied force with estimated linear fits

- Bending Stiffness at stop: 173.6Nm/rad
- Bending Stroke: 18.9mrad

### 5.3 Bending stiffness and bending stroke of all the flexible joints

Now, let's estimate the bending stiffness and stroke for all the flexible joints.

The results are summarized in Table 5.1 for the X direction and in Table 5.2 for the Y direction.

**Table 5.1:** Measured characteristics of the flexible joints in the X direction

	$R_{R_x}$ [Nm/rad]	$k_{R_x,s}$ [Nm/rad]	$R_{x,max}$ [mrad]
1	5.5	173.6	18.9
2	6.1	195.0	17.6
3	6.1	191.3	17.7
4	5.8	136.7	18.3
5	5.7	88.9	22.0
6	5.7	183.9	18.7
7	5.7	157.9	17.9
8	5.8	166.1	17.9
9	5.8	159.5	18.2
10	6.0	143.6	18.1
11	5.0	163.8	17.7
12	6.1	111.9	17.0
13	6.0	142.0	17.4
14	5.8	130.1	17.9
15	5.7	170.7	18.6
16	6.0	148.7	17.5

### 5.4 Analysis

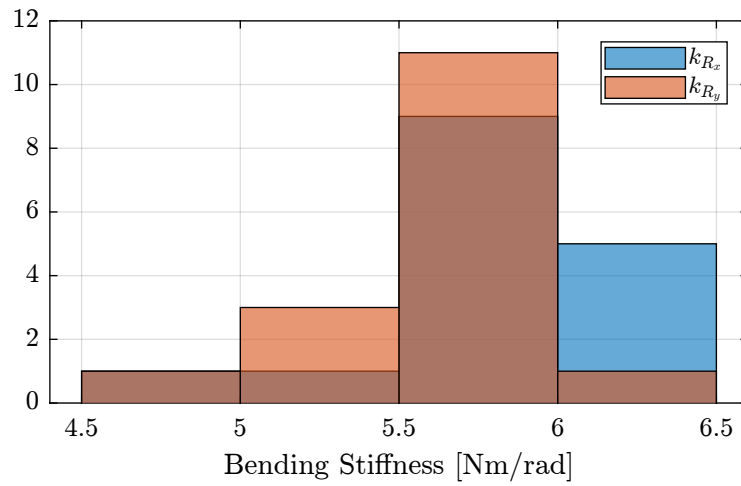
The dispersion of the measured bending stiffness is shown in Figure 5.7 and of the bending stroke in Figure 5.8.

The relation between the measured beam thickness and the measured bending stiffness is shown in Figure 5.9.

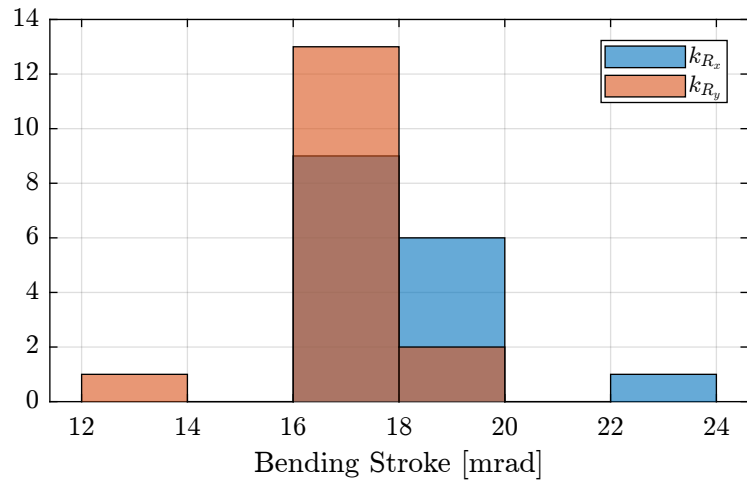


**Table 5.2:** Measured characteristics of the flexible joints in the Y direction

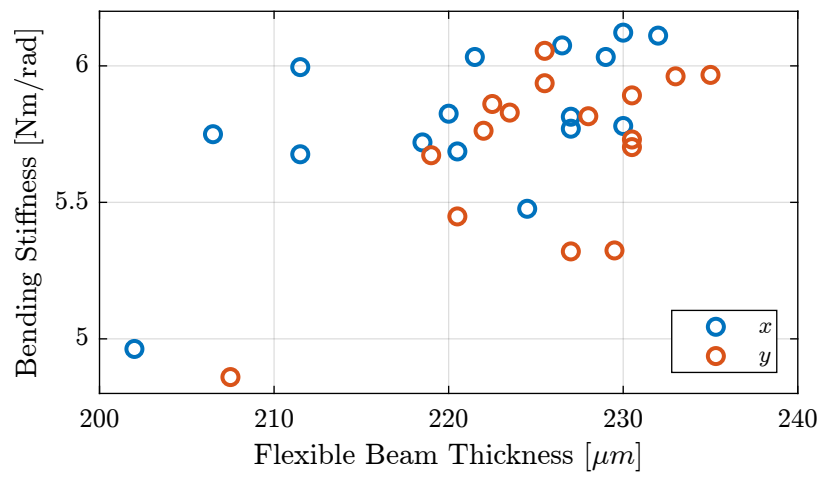
	$R_{R_y}$ [Nm/rad]	$k_{R_{y,s}}$ [Nm/rad]	$R_{y,max}$ [mrad]
1	5.7	323.5	17.9
2	5.9	306.0	17.2
3	6.0	224.4	16.8
4	5.7	247.3	17.8
5	5.8	250.9	13.0
6	5.8	244.5	17.8
7	5.3	214.8	18.1
8	5.8	217.2	17.6
9	5.7	225.0	17.6
10	6.0	254.7	17.3
11	4.9	261.1	18.4
12	5.9	161.5	16.7
13	6.1	227.6	16.8
14	5.9	221.3	17.8
15	5.4	241.5	17.8
16	5.3	291.1	17.7



**Figure 5.7:** Histogram of the measured bending stiffness



**Figure 5.8:** Histogram of the measured bending stroke



**Figure 5.9:** Measured bending stiffness as a function of the estimated flexible beam thickness

## 5.5 Conclusion

### Important

The measured bending stiffness and bending stroke of the flexible joints are very close to the estimated one using a Finite Element Model.

The characteristics of all the flexible joints are also quite close to each other. This should allow us to model them with unique parameters.

## 6 Conclusion