Test Bench - Amplified Piezoelectric Actuator

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Contents

The first goal is to characterize the APA300ML in terms of:

- The, geometric features, electrical capacitance, stroke, hysteresis, spurious resonances. This is performed in Section 2.
- The dynamics from the generated DAC voltage (going to the voltage amplifiers and then applied on the actuator stacks) to the induced displacement, and to the measured voltage by the force sensor stack. Also the "actuator constant" and "sensor constant" are identified. This is done in Section 3.
- Compare the measurements with the Simscape models (2DoF, Super-Element) in order to tuned/-validate the models. This is explained in Section 4.

Table 1: Report sections and corresponding Matlab files

Sections	**Matlab File**
Section ??_	`test_bench_apa_1_.m`

1 Model of the Amplified Piezoelectric Actuator

The Amplified Piezoelectric Actuator (APA) used is the APA300ML from Cedrat technologies (Figure 1.1).

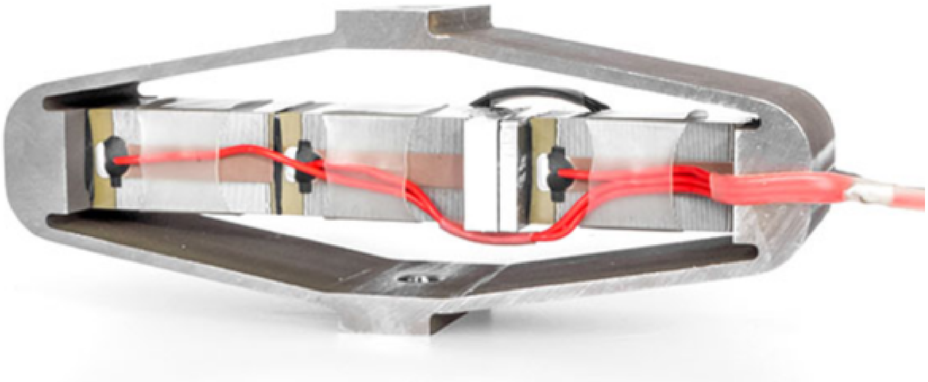

Figure 1.1: Picture of the APA300ML

Two simscape models of the APA300ML are developed:

- Section 1.1: a simple 2 degrees of freedom (DoF) model
- Section 1.2: a "flexible" model using a "super-element" extracted from a Finite Element Model of the APA

For both models, an "actuator constant" and a "sensor constant" are used. These constants are used to link the electrical domain and the mechanical domain. They are described in Section 1.3.

1.1 Two Degrees of Freedom Model

The presented model is based on **souleille18˙concep˙activ˙mount˙space˙applic** and represented in Figure 1.2.

The parameters are described in Table 1.1.

The model is shown again in Figure 1.3. As will be shown in the next section, such model can be quite accurate in modelling the axial behavior of the APA. However, it does not model the flexibility of the APA in the other directions.

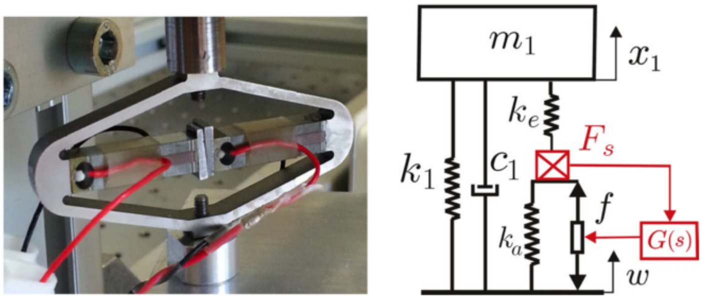

Figure 1.2: Picture of an APA100M from Cedrat Technologies. Simplified model of a one DoF payload mounted on such isolator

Table 1.1: Parameters used for the model of the APA 100M

	Meaning
k_e	Stiffness used to adjust the pole of the isolator
k_1	Stiffness of the metallic suspension when the stack is removed
k_a	Stiffness of the actuator
c_1	Added viscous damping

Therefore this model can be useful for quick simulations as it contains a very limited number of states, but when more complex dynamics of the APA is to be modelled, a flexible model will be used.

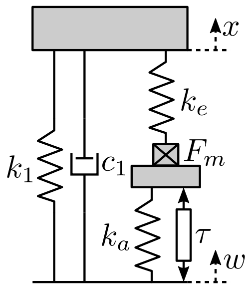

Figure 1.3: Schematic of the 2DoF model for the Amplified Piezoelectric Actuator

1.2 Flexible Model

In order to model with high accuracy the behavior of the APA, a flexible model can be used.

The idea is to do a Finite element model of the structure, and to defined "remote points" as shown in Figure 1.4. Then, on the finite element software, a "super-element" can be extracted which consists of a mass matrix, a stiffness matrix, and the coordinates of the remote points.

This "super-element" can then be included in the Simscape model as shown in Figure 1.5. The remotes points are defined as "frames" in Simscape, and the "super-element" can be connected with other Simscape elements (mechanical joints, masses, force actuators, etc..).

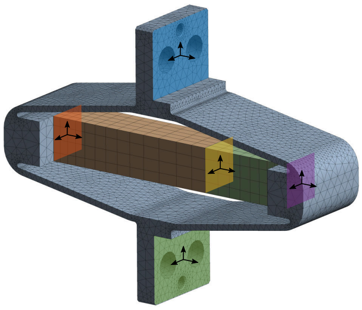

Figure 1.4: Remote points for the APA300ML (Ansys)

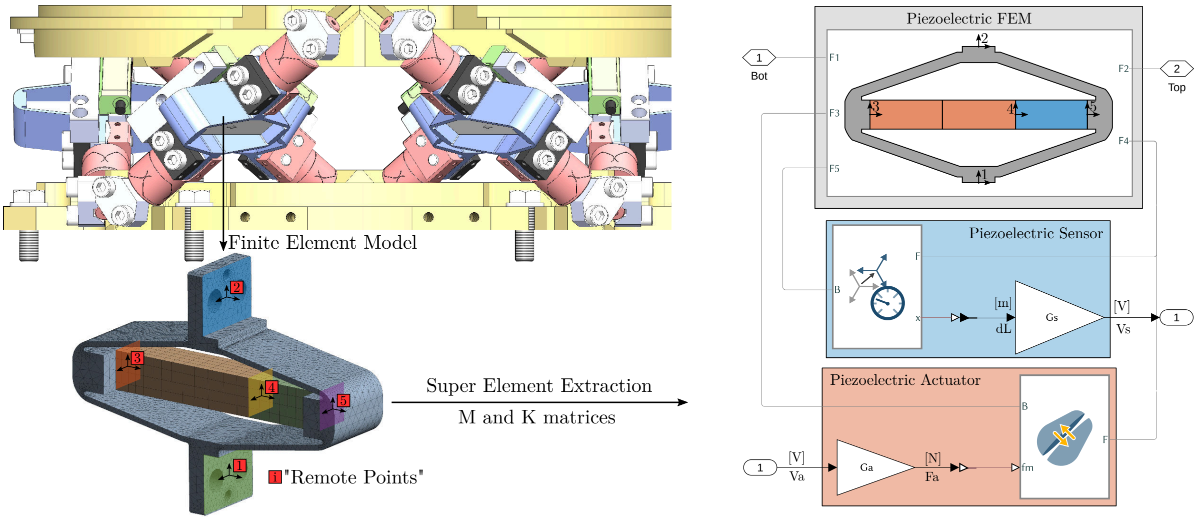

Figure 1.5: From a finite Element Model (Ansys, bottom left) is extract the mass and stiffness matrices that are then used on Simscape (right)

1.3 Actuator and Sensor constants

On Simscape, we want to model both the actuator stacks and the sensors stack. We therefore need to link the electrical domain (voltages, charges) with the mechanical domain (forces, strain). To do so, we use the "actuator constant" and the "sensor constant".

Consider a schematic of the Amplified Piezoelectric Actuator in Figure 1.6.

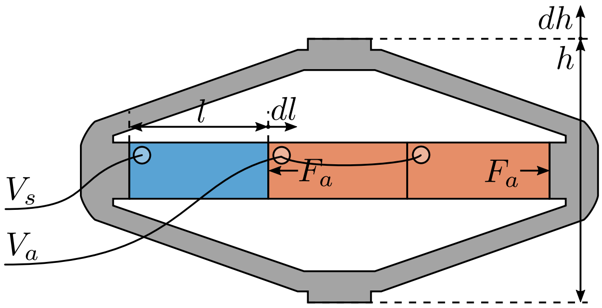

Figure 1.6: Amplified Piezoelectric Actuator Schematic

A voltage V_a applied to the actuator stacks will induce an actuator force F_a:

$$\boxed{F_a = g_a \cdot V_a} \tag{1.1}$$

A change of length dl of the sensor stack will induce a voltage V_s:

$$\boxed{V_s = g_s \cdot dl} \tag{1.2}$$

The block-diagram model of the piezoelectric actuator is then as shown in Figure 1.7.

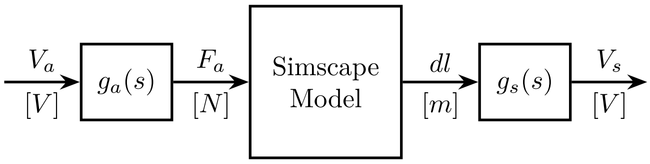

Figure 1.7: Model of the APA with Simscape/Simulink

The constants g_a and g_s will be experimentally estimated.

2 First Basic Measurements

Before using the measurement bench to characterize the APA300ML, first simple measurements are performed:

- Section 2.1: the geometric tolerances of the interface planes are checked
- Section 2.2: the capacitance of the stacks are measured
- Section 2.3: the stroke of the APA are measured
- Section 2.4: the "spurious" resonances of the APA are investigated
- Section **??**: the "spurious" resonances of the struts are measured and compared with the FEM

2.1 Geometrical Measurements

The received APA are shown in Figure 2.1.

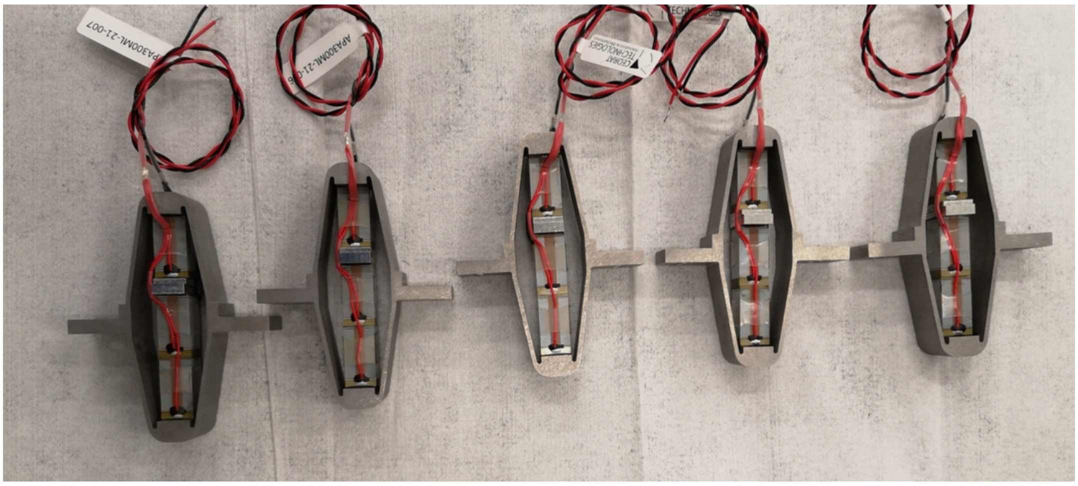

Figure 2.1: Received APA

2.1.1 Measurement Setup

The flatness corresponding to the two interface planes are measured as shown in Figure 2.2.

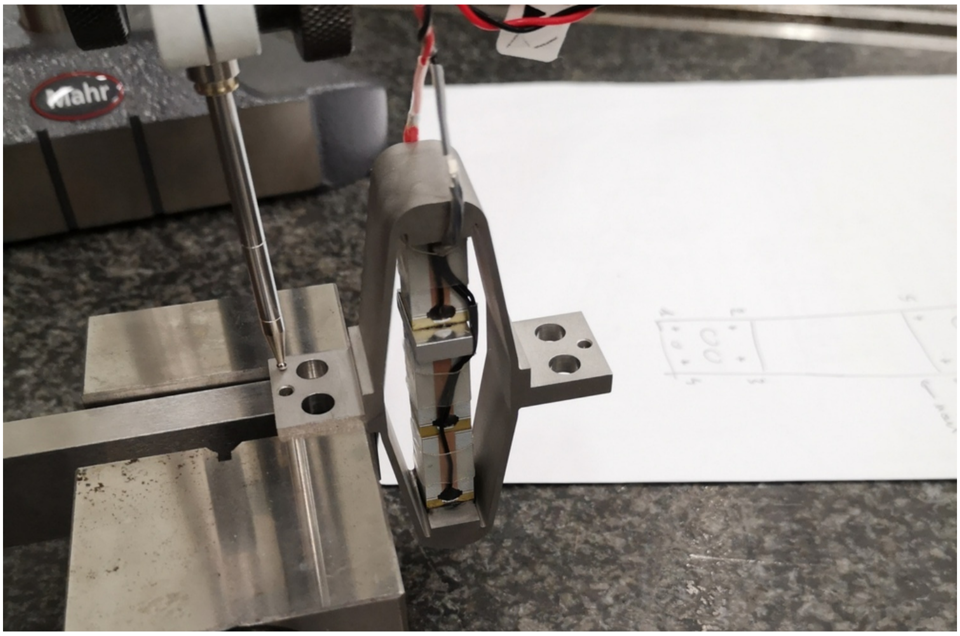

Figure 2.2: Measurement Setup

2.1.2 Measurement Results

The height (Z) measurements at the 8 locations (4 points by plane) are defined below. The X/Y Positions of the 8 measurement points are defined below. Finally, the flatness is estimated by fitting a plane through the 8 points using the `fminsearch` command. The obtained flatness are shown in Table 2.1.

Table 2.1: Estimated flatness

	Flatness [μm]
APA 1	8.9
APA 2	3.1
APA 3	9.1
APA 4	3.0
APA 5	1.9
APA 6	7.1
APA 7	18.7

Important

The measured flatness of the APA300ML interface planes are within the specifications.

2.2 Electrical Measurements

2.2.1 Measurement Setup

Note

The capacitance of the stacks is measure with the LCR-800 Meter (doc) shown in Figure 2.3. The excitation frequency is set to be 1kHz.

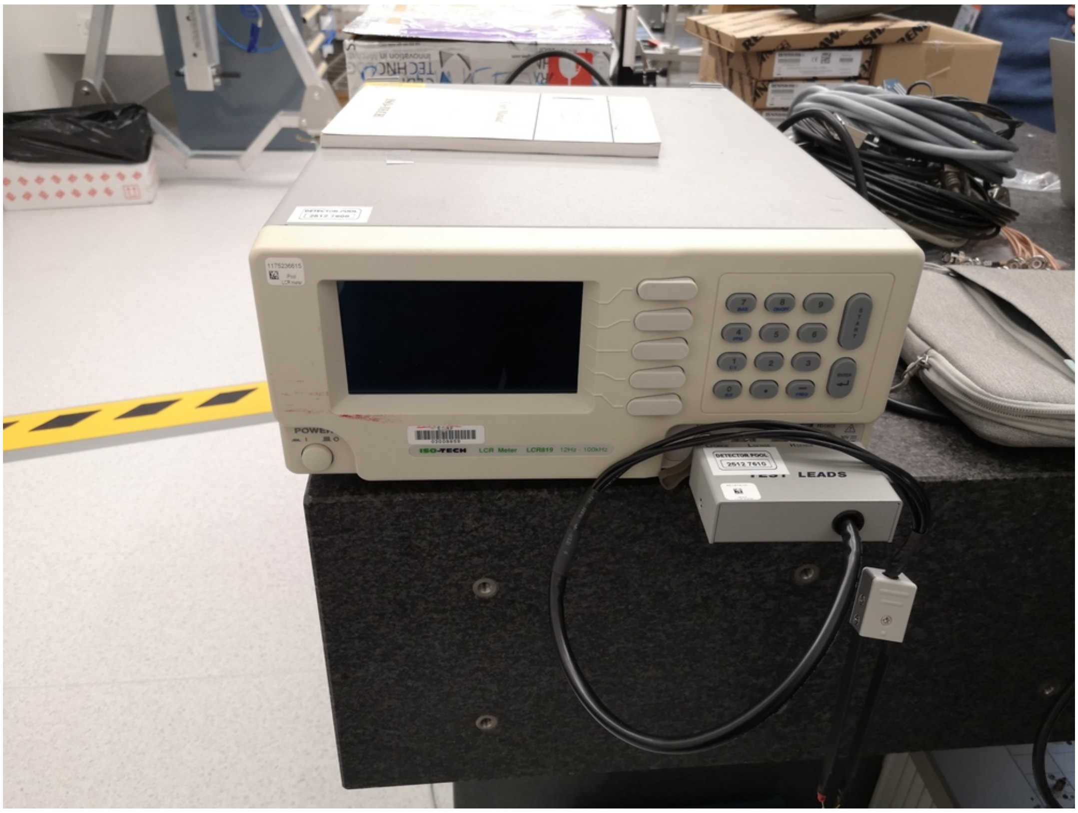

Figure 2.3: LCR Meter used for the measurements

2.2.2 Measured Capacitance

From the documentation of the APA300ML, the total capacitance of the three stacks should be between $18\mu F$ and $26\mu F$ with a nominal capacitance of $20\mu F$. However, from the documentation of the stack themselves, it can be seen that the capacitance of a single stack should be $4.4\mu F$. Clearly, the total capacitance of the APA300ML if more than just three times the capacitance of one stack.

Question

Could it be possible that the capacitance of the stacks increase that much when they are pre-stressed?

The measured capacitance of the stacks are summarized in Table 2.2.

Table 2.2: Capacitance measured with the LCR meter. The excitation signal is a sinus at 1kHz

	Sensor Stack	Actuator Stacks
APA 1	5.10	10.03
APA 2	4.99	9.85
APA 3	1.72	5.18
APA 4	4.94	9.82
APA 5	4.90	9.66
APA 6	4.99	9.91
APA 7	4.85	9.85

Important

From the measurements (Table 2.2), the capacitance of one stack is found to be $\approx 5 \mu F$.

Warning

There is clearly a problem with APA300ML number 3 The APA number 3 has ben sent back to Cedrat, and a new APA300ML has been shipped back.

2.3 Stroke measurement

We here wish to estimate the stroke of the APA.

To do so, one side of the APA is fixed, and a displacement probe is located on the other side as shown in Figure 2.4.

Then, a voltage is applied on either one or two stacks using a DAC and a voltage amplifier.

Note

Here are the documentation of the equipment used for this test bench:

- **Voltage Amplifier**: PD200 with a gain of 20
- **16bits DAC**: IO313 Speedgoat card
- **Displacement Probe**: Millimar C1216 electronics and Millimar 1318 probe

From the documentation, the nominal stroke of the APA300ML is $304\, \mu m$.

2.3.1 Voltage applied on one stack

Let's first look at the relation between the voltage applied to **one** stack to the displacement of the APA as measured by the displacement probe.

The applied voltage is shown in Figure 2.5.

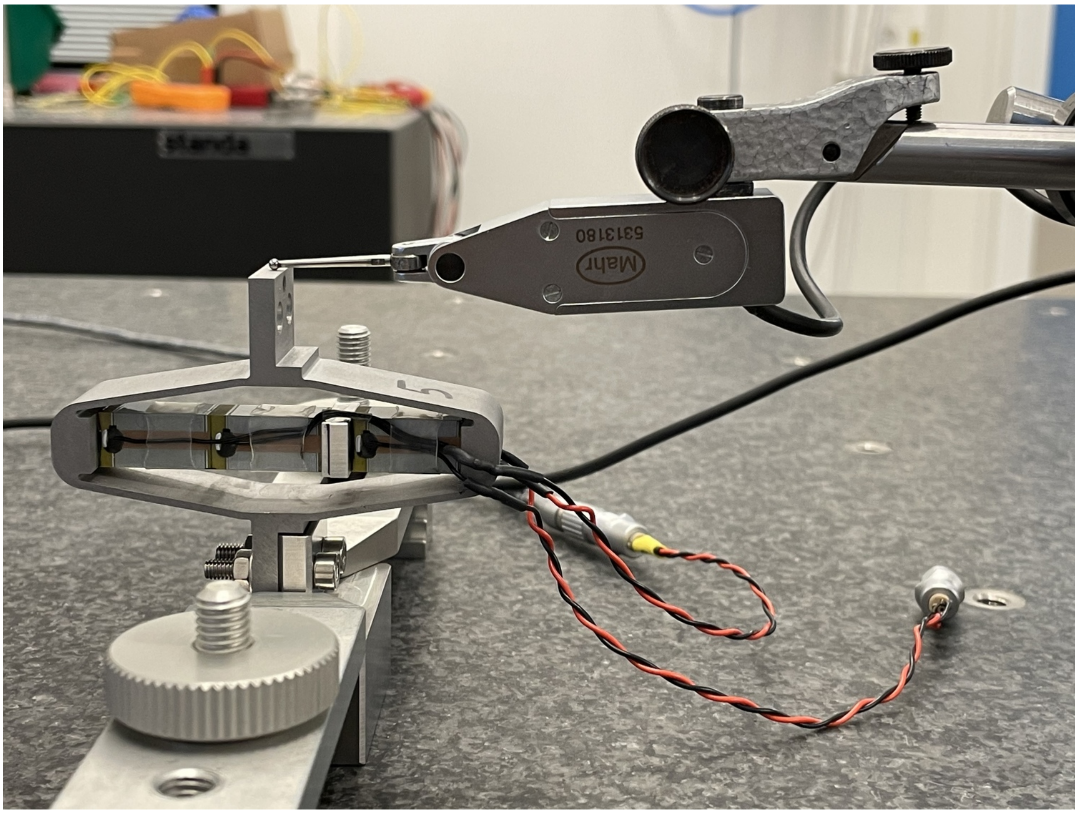

Figure 2.4: Bench to measured the APA stroke

figs/apa_stroke_voltage_time.pdf

Figure 2.5: Applied voltage as a function of time

The obtained displacements for all the APA are shown in Figure 2.6. The displacement is set to zero at initial time when the voltage applied is -20V.

figs/apa_stroke_time_1s.pdf

Figure 2.6: Displacement as a function of time for all the APA300ML (only one stack is used as an actuator)

Finally, the displacement is shown as a function of the applied voltage in Figure 2.7. We can clearly see that there is a problem with the APA 3. Also, there is a large hysteresis.

figs/apa_d_vs_V_1s.pdf

Figure 2.7: Displacement as a function of the applied voltage (on only one stack)

Important

We can clearly confirm from Figure 2.7 that there is a problem with the APA number 3.

2.3.2 Voltage applied on two stacks

Now look at the relation between the voltage applied to the **two** other stacks to the displacement of the APA as measured by the displacement probe.

The obtained displacement is shown in Figure 2.8. The displacement is set to zero at initial time when the voltage applied is -20V.

figs/apa_stroke_time_2s.pdf

Figure 2.8: Displacement as a function of time for all the APA300ML (two stacks are used as actuators)

Finally, the displacement is shown as a function of the applied voltage in Figure 2.9.

figs/apa_d_vs_V_2s.pdf

Figure 2.9: Displacement as a function of the applied voltage on two stacks

2.3.3 Voltage applied on all three stacks

Finally, we can combine the two measurements to estimate the relation between the displacement and the voltage applied to the **three** stacks (Figure 2.10).

figs/apa_d_vs_V_3s.pdf

Figure 2.10: Displacement as a function of the applied voltage on all three stacks

The obtained maximum stroke for all the APA are summarized in Table 2.3.

Table 2.3: Measured maximum stroke

	Stroke [μm]
APA 1	373.2
APA 2	365.5
APA 3	181.7
APA 4	359.7
APA 5	361.5
APA 6	363.9
APA 7	358.4

2.3.4 Conclusion

Important

The except from APA 3 that has a problem, all the APA are similar when it comes to stroke and hysteresis. Also, the obtained stroke is more than specified in the documentation. Therefore, only two stacks can be used as an actuator.

2.4 Spurious resonances - APA

2.4.1 Introduction

From a Finite Element Model of the struts, it have been found that three main resonances are foreseen to be problematic for the control of the APA300ML (Figure 2.11):

- Mode in X-bending at 189Hz
- Mode in Y-bending at 285Hz
- Mode in Z-torsion at 400Hz

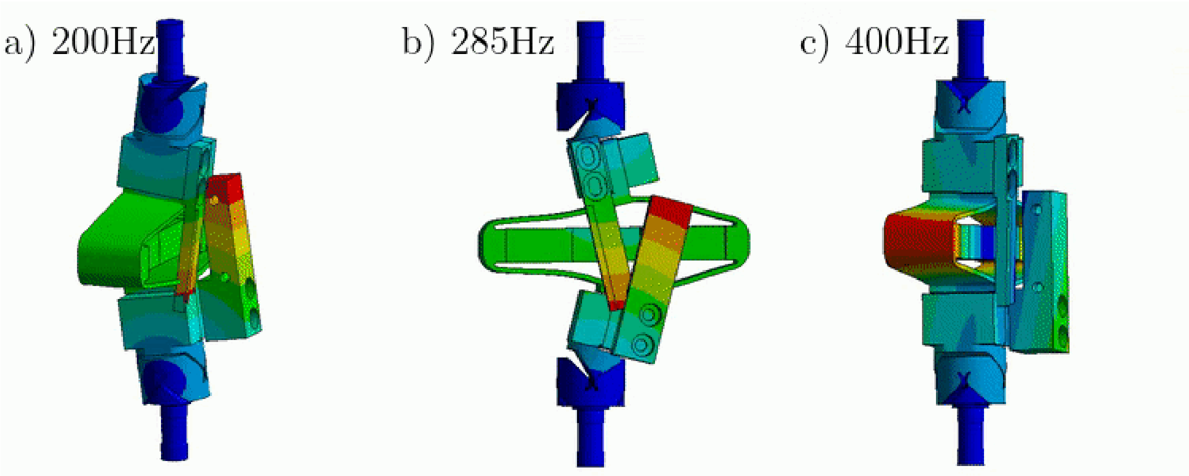

Figure 2.11: Spurious resonances. a) X-bending mode at 189Hz. b) Y-bending mode at 285Hz. c) Z-torsion mode at 400Hz

These modes are present when flexible joints are fixed to the ends of the APA300ML.

In this section, we try to find the resonance frequency of these modes when one end of the APA is fixed and the other is free.

In the section **??**, a similar measurement will be performed directly on the struts.

2.4.2 Measurement Setup

The measurement setup is shown in Figure 2.12. A Laser vibrometer is measuring the difference of motion between two points. The APA is excited with an instrumented hammer and the transfer function from the hammer to the measured rotation is computed.

Note

The instrumentation used are:

- Laser Doppler Vibrometer Polytec OFV512

- Instrumented hammer

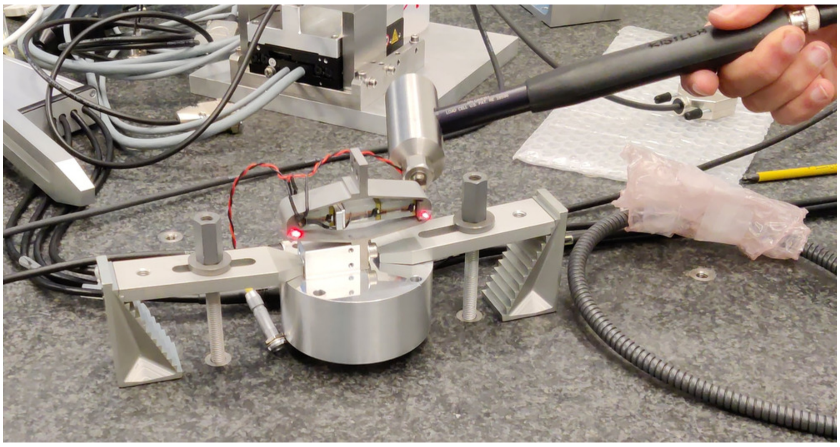

Figure 2.12: Measurement setup with a Laser Doppler Vibrometer and one instrumental hammer

2.4.3 X-Bending Mode

The vibrometer is setup to measure the X-bending motion is shown in Figure 2.13. The APA is excited with an instrumented hammer having a solid metallic tip. The impact point is on the back-side of the APA aligned with the top measurement point.

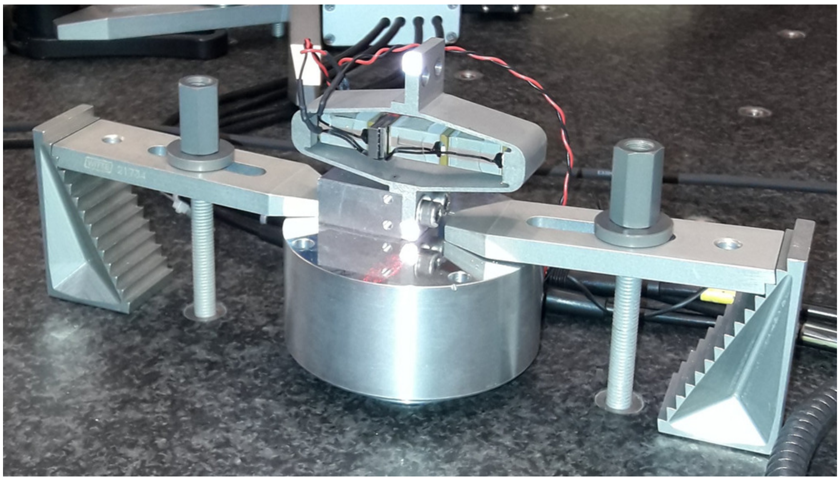

Figure 2.13: X-Bending measurement setup

The data is loaded. The configuration (Sampling time and windows) for `tfestimate` is done: The transfer function from the input force to the output "rotation" (difference between the two measured distances). The result is shown in Figure 2.14.

The can clearly observe a nice peak at 280Hz, and then peaks at the odd "harmonics" (third "harmonic" at 840Hz, and fifth "harmonic" at 1400Hz).

Then the APA is in the "free-free" condition, this bending mode is foreseen to be at 200Hz (Figure 2.11). We are here in the "fixed-free" condition. If we consider that we therefore double the stiffness figs/apa300ml_meas_freq_bending_x.pdf

Figure 2.14: Obtained FRF for the X-bending

associated with this mode, we should obtain a resonance a factor $\sqrt{2}$ higher than 200Hz which is indeed 280Hz. Not sure this reasoning is correct though.

2.4.4 Y-Bending Mode

The setup to measure the Y-bending is shown in Figure 2.15.

The impact point of the instrumented hammer is located on the back surface of the top interface (on the back of the 2 measurements points).

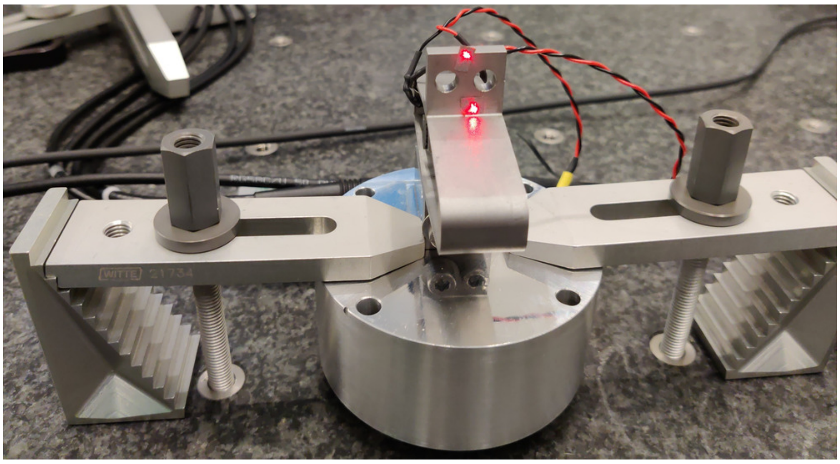

Figure 2.15: Y-Bending measurement setup

The data is loaded, and the transfer function from the force to the measured rotation is computed. The results are shown in Figure 2.16. The main resonance is at 412Hz, and we also see the third "harmonic" at 1220Hz.

figs/apa300ml_meas_freq_bending_y.pdf

Figure 2.16: Obtained FRF for the Y-bending

We can apply the same reasoning as in the previous section and estimate the mode to be a factor $\sqrt{2}$

higher than the mode estimated in the "free-free" condition. We would obtain a mode at 403Hz which is very close to the one estimated here.

2.4.5 Z-Torsion Mode

Finally, we measure the Z-torsion resonance as shown in Figure 2.17.

The excitation is shown on the other side of the APA, on the side to excite the torsion motion.

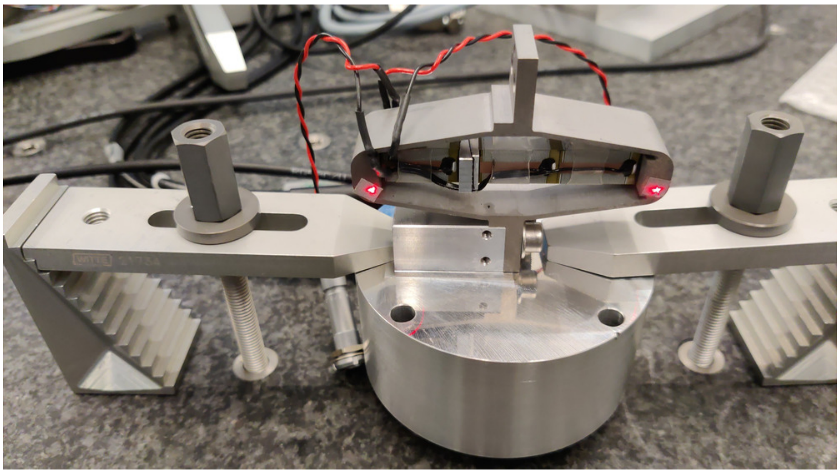

Figure 2.17: Z-Torsion measurement setup

The data is loaded, and the transfer function computed. The results are shown in Figure 2.18. We observe a first peak at 267Hz, which corresponds to the X-bending mode that was measured at 280Hz. And then a second peak at 415Hz, which corresponds to the X-bending mode that was measured at 412Hz. A third mode at 800Hz could correspond to this torsion mode.

figs/apa300ml_meas_freq_torsion_z.pdf

Figure 2.18: Obtained FRF for the Z-torsion

In order to verify that, the APA is excited on the top part such that the torsion mode should not be excited. The two FRF are compared in Figure 2.19. It is clear that the first two modes does not correspond to the torsional mode. Maybe the resonance at 800Hz, or even higher resonances. It is difficult to conclude here.

2.4.6 Compare

The three measurements are shown in Figure 2.20.

figs/apa300ml_meas_freq_torsion_z_comp.pdf

Figure 2.19: Obtained FRF for the Z-torsion

figs/apa300ml_meas_freq_compare.pdf

Figure 2.20: Obtained FRF - Comparison

2.4.7 Conclusion

When two flexible joints are fixed at each ends of the APA, the APA is mostly in a free/free condition in terms of bending/torsion (the bending/torsional stiffness of the joints being very small).

In the current tests, the APA are in a fixed/free condition. Therefore, it is quite obvious that we measured higher resonance frequencies than what is foreseen for the struts. It is however quite interesting that there is a factor $\approx \sqrt{2}$ between the two (increased of the stiffness by a factor 2?).

Table 2.4: Measured frequency of the modes

Mode	**FEM - Strut mode**	**Measured Frequency**
X-Bending	189Hz	280Hz
Y-Bending	285Hz	410Hz
Z-Torsion	400Hz	800Hz?

3 Dynamical measurements - APA

In this section, a measurement test bench is used to extract all the important parameters of the Amplified Piezoelectric Actuator APA300ML.

This include:

- Stroke
- Stiffness
- Hysteresis
- "Actuator constant": Gain from the applied voltage V_a to the generated Force F_a
- "Sensor constant": Gain from the sensor stack strain δL to the generated voltage V_s
- Dynamical behavior from the actuator to the force sensor and to the motion of the APA

The bench is shown in Figure 3.1, and a zoom picture on the APA and encoder is shown in Figure 3.2.

The bench is schematically shown in Figure 3.3 and the signal used are summarized in Table 3.1.

Table 3.1: Variables used during the measurements

Variable	Description	Unit	Hardware
`Va`	Output DAC voltage	[V]	DAC - Ch. 1 - PD200 - APA
`Vs`	Measured stack voltage (ADC)	[V]	APA - ADC - Ch. 1
`de`	Encoder Measurement	[m]	PEPU Ch. 1 - IO318(1) Ch. 1
`da`	Attocube Measurement	[m]	PEPU Ch. 2 - IO318(1) Ch. 2
`t`	Time	[s]	

This section is structured as follows:

- Section 3.1: the measurements are first performed on one APA.
- Section 3.2: the same measurements are performed on all the APA and are compared.

3.1 Measurements on APA 1

Measurements are first performed on only **one** APA. Once the measurement procedure is validated, it is performed on all the other APA.

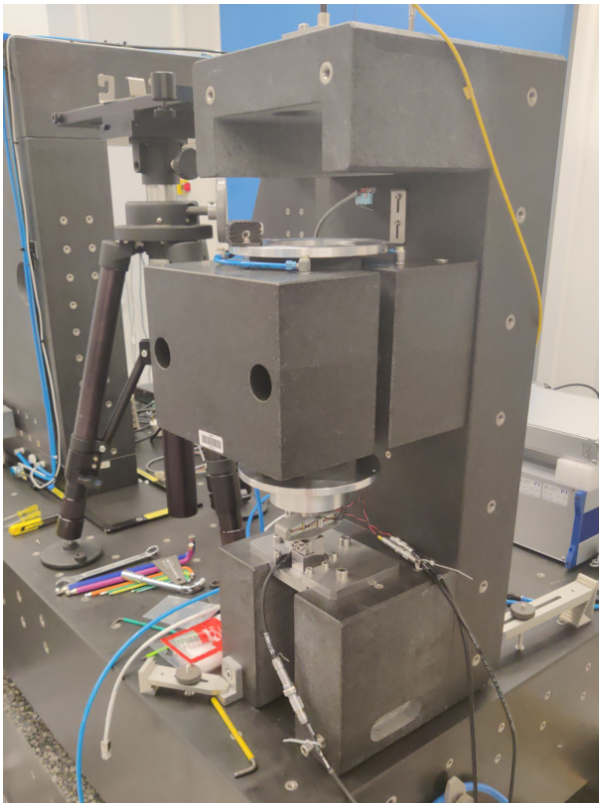

Figure 3.1: Picture of the test bench

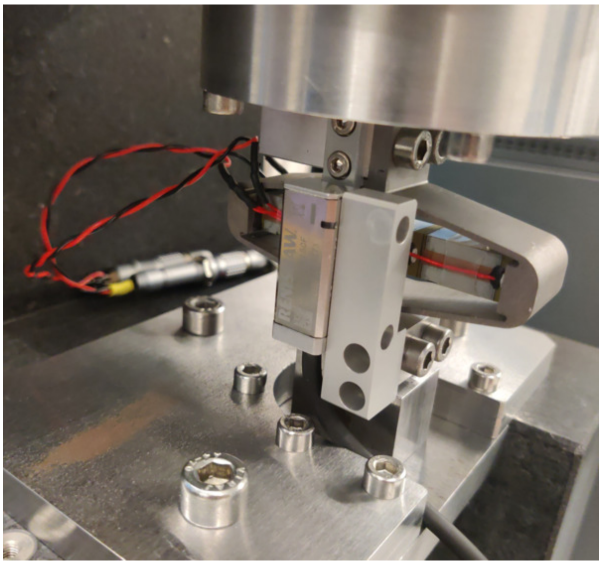

Figure 3.2: Zoom on the APA with the encoder

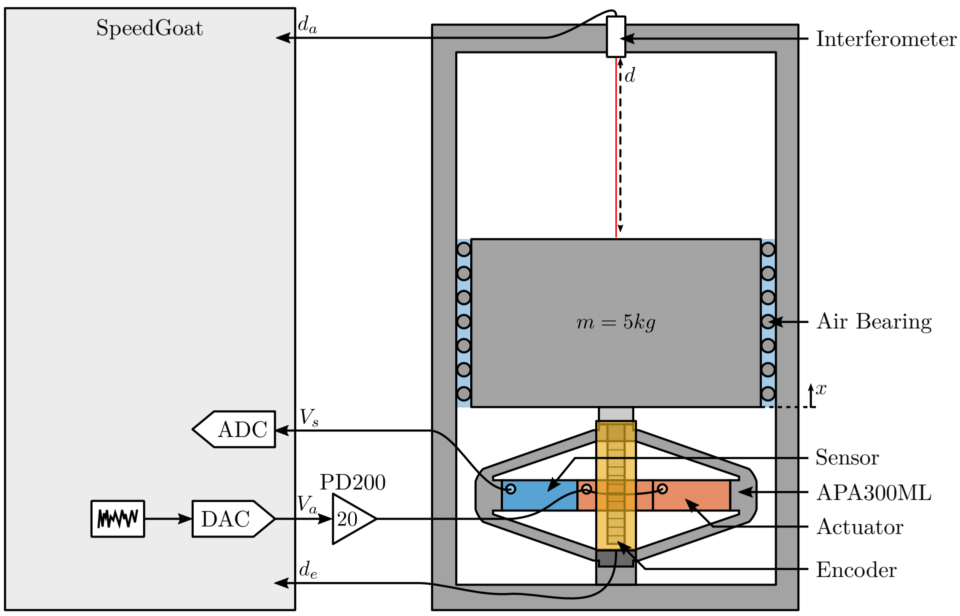

Figure 3.3: Schematic of the Test Bench

3.1.1 Excitation Signals

Different excitation signals are used to perform FRF estimations.

Typically, this is done in three steps:

1. A low pass filtered white noise is used with rather small amplitudes (Figure 3.4). This first excitation is used to estimate the main resonance of the system.

2. A sweep-sine from 10Hz to 400Hz is used (Figure 3.5). The sweep-sine is is notched around the estimated resonance of the system.

3. A band-limited white noise from 300Hz to 2kHz is used to estimate the high frequency behavior (Figure 3.6).

For all the excitation signals, before the excitation starts, the mean voltage is slowly increased halfway between the minimum voltage (-20V) and the maximum (150V).

The first measurement is only used to have a first estimation of the dynamics and verify that everything is setup correctly. The second excitation is done to estimate the dynamics from 10Hz to 350Hz and the third excitation from 350Hz to 2kHz. The second and third measurements are therefore combined in the frequency domain to form one good estimation of the dynamics from 10Hz up to 2kHz.

3.1.2 First Measurement

For this first measurement for the first APA, a basic logarithmic sweep is used between 10Hz and 2kHz.

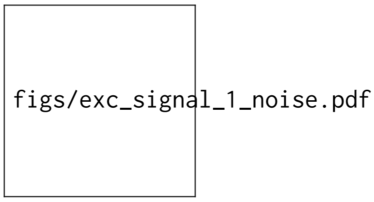

Figure 3.4: Low pass filtered white noise. Time domain (left), Frequency domain (right)

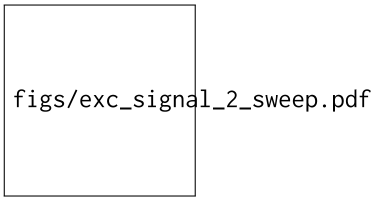

Figure 3.5: Sweep Sine with a decreased amplitude around the resonance of the APA

The data are loaded. The initial time is set to zero. The excitation signal is shown in Figure 3.7. It is a sweep sine from 10Hz up to 2kHz filtered with a notch centered with the main resonance of the system and a low pass filter.

3.1.3 FRF - Setup

Let's define the sampling time/frequency. Then we defined a "Hanning" windows that will be used for the spectral analysis: We get the frequency vector that will be the same for all the frequency domain analysis.

3.1.4 FRF - Encoder and Interferometer

In this section, the transfer function from the excitation voltage V_a to the encoder measured displacement d_e and interferometer measurement d_a.

The coherence from V_a to d_e and from V_a to d_a are computed and shown in Figure 3.8. They are quite good from 10Hz up to 500Hz.

The transfer functions are then estimated and shown in Figure 3.9. It is shown than both the encoder and interferometers are measuring the same dynamics up to $\approx 700\,Hz$. Above that, it is possible that

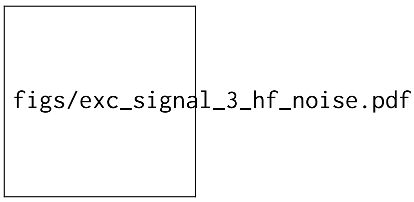

Figure 3.6: Band-pass white noise. Time domain (left), Frequency domain (right)

figs/apa_bench_exc_sweep.pdf

Figure 3.7: Excitation voltage

figs/apa_1_coh_dvf.pdf

Figure 3.8: Coherence for the identification from V_a to d_e

there is some flexible elements apart from the APA that is adding resonances into one or the other FRF.

figs/apa_1_frf_dvf.pdf

Figure 3.9: Obtained transfer functions from V_a to both d_e and d_a

Important

The transfer functions obtained in Figure 3.9 are very close to what was expected:

- constant gain at low frequency
- resonance at around 100Hz which corresponds to the APA axial mode
- no further resonance up until high frequency ($\approx 700\,Hz$) at which points several elements of the test bench can induces resonances in the measured FRF

However, it was not expected to observe a "double resonance" at around 95Hz (instead of only one resonance).

3.1.5 FRF - Force Sensor

Now the dynamics from excitation voltage V_a to the force sensor stack voltage V_s is identified.

The coherence is computed and shown in Figure 3.10 and found very good from 10Hz up to 2kHz.

The transfer function is estimated and shown in Figure 3.11.

figs/apa_1_coh_iff.pdf

Figure 3.10: Coherence for the identification from V_a to V_s

figs/apa_1_frf_iff.pdf

Figure 3.11: Obtained transfer functions from V_a to V_s

Important

The obtained dynamics from the excitation voltage V_a to the measured sensor stack voltage V_s is corresponding to what was expected:

- constant gain at low frequency
- complex conjugate zero and then complex conjugate pole
- constant gain at high frequency

3.1.6 Hysteresis

We here wish to visually see the amount of hysteresis present in the APA.

To do so, a quasi static sinusoidal excitation V_a at different voltages is used.

The offset is 65V (halfway between -20V and 150V), and the sin amplitude is ranging from 1V up to 80V (full range).

For each excitation amplitude, the vertical displacement d of the mass is measured.

Then, d is plotted as a function of V_a for all the amplitudes.

We expect to obtained something like the hysteresis shown in Figure 3.12.

The data is loaded. The excitation voltage amplitudes are: The excitation voltage and the measured displacement are shown in Figure 3.13.

For each amplitude, we only take the last sinus in order to reduce possible transients. Also, the motion is centered on zero.

The measured displacement at a function of the output voltage are shown in Figure 3.14.

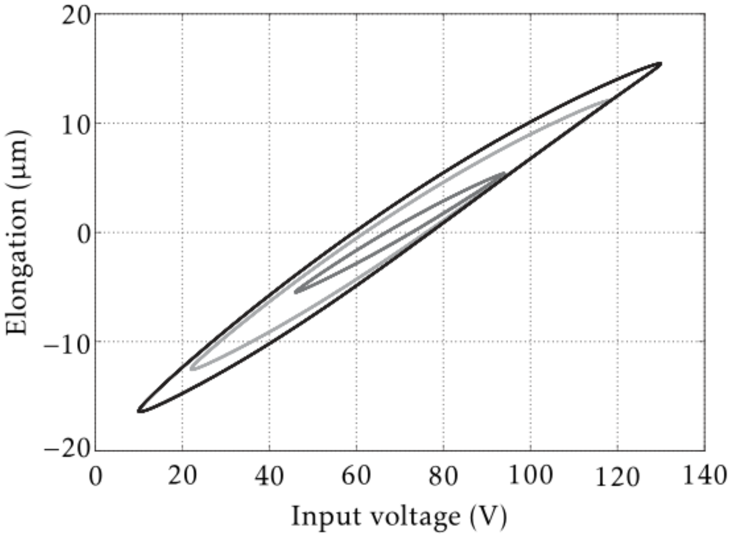

Figure 6.16: *Measured hysteresis loops for a single PE actuator, 70 V bias + 1 Hz sine wave: 60 V amplitude (black), 48 V amplitude (light grey), 24 V amplitude (dark grey)*

Figure 3.12: Expected Hysteresis **poel10˙explor˙activ˙hard˙mount˙vibrat**

figs/hyst_exc_signal_time.pdf

Figure 3.13: Excitation voltage and measured displacement

figs/hyst_results_multi_ampl.pdf

Figure 3.14: Obtained hysteresis for multiple excitation amplitudes

Important

From Figure 3.14, it is quite clear that hysteresis is increasing with the excitation amplitude. For small excitation amplitudes ($V_a < 0.4\,V$) the hysteresis stays reasonably small.
Also, it is quite interesting to see that no hysteresis is found on the sensor stack voltage when using the same excitation signal.

3.1.7 Estimation of the APA axial stiffness

In order to estimate the stiffness of the APA, a weight with known mass m_a is added on top of the suspended granite and the deflection d_e is measured using the encoder.

The APA stiffness can then be estimated to be:

$$k_{\text{apa}} = \frac{m_a g}{d} \tag{3.1}$$

The data is loaded, and the measured displacement is shown in Figure 3.15.

figs/apa_1_meas_stiffness.pdf

Figure 3.15: Measured displacement when adding the mass and removing the mass

From Figure 3.15, it can be seen that there are some drifts that are probably due to some creep. This will induce some uncertainties in the measured stiffness.

Here, a mass of 6.4 kg was used: The stiffness is then computed as follows: And the stiffness obtained is very close to the one specified in the documentation ($k = 1.794\,[N/\mu m]$).

```
Results
k = 1.68 [N/um]
```

The stiffness could also be estimated based on the main vertical resonance of the system at $\omega_z = 2\pi \cdot 94\,[rad/s]$. The suspended mass is $m_{\text{sus}} = 5\,kg$. And therefore, the axial stiffness of the APA can be estimated to be:

$$k_{\text{APA}} = m_{\text{sus}} \omega_z^2 \tag{3.2}$$

```
Results
k = 1.99 [N/um]
```

The two values are found relatively close to each other. Anyway, the stiffness of the model will be tuned to match the measured FRF.

3.1.8 Stiffness change due to electrical connections

Changes in the electrical impedance connected to the piezoelectric actuator causes changes in the mechanical compliance (or stiffness) of the piezoelectric actuator.

In this section is measured the stiffness of the APA whether the piezoelectric actuator is connected to an open circuit or a short circuit (e.g. the output of a voltage amplifier).

Note here that the resistor in parallel to the sensor stack is present in both cases.

First, the data are loaded. And the initial displacement is set to zero. The measured displacements are shown in Figure 3.16.

figs/apa_meas_k_time_oc_cc.pdf

Figure 3.16: Measured displacement

And the stiffness is estimated in both case. The results are shown in Table 3.2.

Table 3.2: Measured stiffnesses on "open" and "closed" circuits

	$k [N/\mu m]$
Not connected	2.3
Connected	1.7

Important

Clearly, connecting the actuator stacks to the amplified (basically equivalent as to short circuiting them) lowers its stiffness.

3.1.9 Effect of the resistor on the IFF Plant

A resistor $R \approx 80.6\,k\Omega$ is added in parallel with the sensor stack. This has the effect to form a high pass filter with the capacitance of the stack.

This is done for two reasons (explained in details this document):

1. Limit the voltage offset due to the input bias current of the ADC
2. Limit the low frequency gain

The (low frequency) transfer function from V_a to V_s with and without this resistor have been measured.

We use a very long "Hanning" window for the spectral analysis in order to estimate the low frequency behavior. And we estimate the transfer function from V_a to V_s in both cases: With the following values of the resistor and capacitance, we obtain a first order high pass filter with a crossover frequency equal to:

Results

```
f0 = 0.39 [Hz]
```

The transfer function of the corresponding high pass filter is: Let's compare the transfer function from actuator stack to sensor stack with and without the added resistor in Figure 3.17.

figs/frf_iff_effect_R.pdf

Figure 3.17: Transfer function from V_a to V_s with and without the resistor k

Important

The added resistor has indeed the expected effect of forming an high pass filter.

3.2 Comparison of all the APA

The same measurements that was performed in Section 3.1 are now performed on all the APA and then compared.

3.2.1 Axial Stiffnesses - Comparison

Let's first compare the APA axial stiffnesses.

The added mass is: Here are the numbers of the APA that have been measured: The data are loaded. The raw measurements are shown in Figure 3.18. All the APA seems to have similar stiffness except the APA 7 which show strange behavior.

Warning

It is however strange that the displacement d_e when the mass is removed is higher for the APA 7 than for the other APA.
It turns out the PD200 amplifier was connected to only one stack, the other stack was open circuited. Therefore, the total axial stiffness of the APA was increased.

The stiffnesses are computed for all the APA and are summarized in Table 3.3.

figs/apa_meas_k_time.pdf

Figure 3.18: Raw measurements for all the APA. A mass of 6.4kg is added at arround 15s and removed at arround 22s

Table 3.3: Measured stiffnesses

APA Num	$k [N/\mu m]$
1	1.68
2	1.69
4	1.7
5	1.7
6	1.7
7	1.93
8	1.73

Important

The APA300ML manual specifies the nominal stiffness to be 1.8 $[N/\mu m]$ which is very close to what have been measured. Only the APA number 7 is a little bit higher, due to the fact that one of the stack was open-circuited instead of short circuited.

3.2.2 FRF - Setup

As the APA7 was not correctly wired, it is ignored: The identification is performed in three steps:

1. White noise excitation with small amplitude. This is used to determine the main resonance of the system.
2. Sweep sine excitation with the amplitude lowered around the resonance. The sweep sine is from 10Hz to 400Hz.
3. High frequency noise. The noise is band-passed between 300Hz and 2kHz.

Then, the result of the second identification is used between 10Hz and 350Hz and the result of the third identification if used between 350Hz and 2kHz.

The data are loaded for both the second and third identification: The time is the same for all measurements. Then we defined a "Hanning" windows that will be used for the spectral analysis: We get the frequency vector that will be the same for all the frequency domain analysis.

3.2.3 FRF - Encoder and Interferometer

In this section, the dynamics from excitation voltage V_a to encoder measured displacement d_e is identified.

We compute the coherence for 2nd and 3rd identification: The coherence is shown in Figure 3.19, and it is found that the coherence is good from low frequency up to 700Hz.

figs/apa_frf_dvf_plant_coh.pdf

Figure 3.19: Obtained coherence for the plant from V_a to d_e

Then, the transfer function from the DAC output voltage V_a to the measured displacement by the encoders is computed: The obtained transfer functions are shown in Figure 3.20. They are all superimposed.

figs/apa_frf_dvf_plant_tf.pdf

Figure 3.20: Estimated FRF for the DVF plant (transfer function from V_a to the encoder d_e)

A zoom on the main resonance is shown in Figure 3.21. It is clear that the responses around the resonances are well matching for all the APA.

It is also clear that there is not a single resonance but two resonances, a first one at 95Hz and a second one at 105Hz.

Question

Why is there a double resonance at around 94Hz?

figs/apa_frf_dvf_zoom_res_plant_tf.pdf

Figure 3.21: Estimated FRF for the DVF plant (transfer function from V_a to the encoder d_e) - Zoom on the main resonance

3.2.4 FRF - Force Sensor

In this section, the dynamics from V_a to V_s is identified.

First the coherence is computed and shown in Figure 3.22. The coherence is very nice from 10Hz to 2kHz. It is only dropping near a zeros at 40Hz, and near the resonance at 95Hz (the excitation amplitude being lowered).

figs/apa_frf_iff_plant_coh.pdf

Figure 3.22: Obtained coherence for the IFF plant

Then the FRF are estimated and shown in Figure 3.23

figs/apa_frf_iff_plant_tf.pdf

Figure 3.23: Identified IFF Plant

3.2.5 Conclusion

Important

So far, all the measured FRF are showing the dynamical behavior that was expected.

4 Test Bench APA300ML - Simscape Model

In this section, a simscape model (Figure 4.1) of the measurement bench is used to compare the model of the APA with the measured FRF.

After the transfer functions are extracted from the model (Section 4.1), the comparison of the obtained dynamics with the measured FRF will permit to:

1. Estimate the "actuator constant" and "sensor constant" (Section 4.2)
2. Tune the model of the APA to match the measured dynamics (Section 4.3)

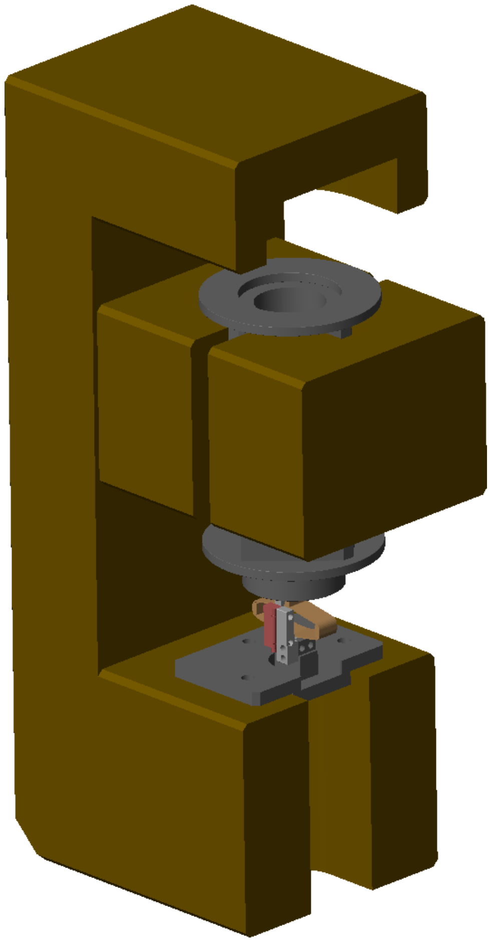

Figure 4.1: Screenshot of the Simscape model

4.1 First Identification

The APA is first initialized with default parameters: The transfer function from excitation voltage V_a (before the amplification of 20 due to the PD200 amplifier) to:

1. the sensor stack voltage V_s
2. the measured displacement by the encoder d_e
3. the measured displacement by the interferometer d_a

The obtain dynamics are shown in Figure 4.2 and 4.3. It can be seen that:

- the shape of these bode plots are very similar to the one measured in Section 3 expect from a change in gain and exact location of poles and zeros
- there is a sign error for the transfer function from V_a to V_s. This will be corrected by taking a negative "sensor gain".
- the low frequency zero of the transfer function from V_a to V_s is minimum phase as expected. The measured FRF are showing non-minimum phase zero, but it is most likely due to measurements artifacts.

figs/apa_model_bench_bode_vs.pdf

Figure 4.2: Bode plot of the transfer function from V_a to V_s

figs/apa_model_bench_bode_dl_z.pdf

Figure 4.3: Bode plot of the transfer function from V_a to d_L and to z

4.2 Identify Sensor/Actuator constants and compare with measured FRF

4.2.1 How to identify these constants?

Piezoelectric Actuator Constant Using the measurement test-bench, it is rather easy the determine the static gain between the applied voltage V_a to the induced displacement d.

$$d = g_{d/V_a} \cdot V_a \tag{4.1}$$

Using the Simscape model of the APA, it is possible to determine the static gain between the actuator force F_a to the induced displacement d:

$$d = g_{d/F_a} \cdot F_a \tag{4.2}$$

From the two gains, it is then easy to determine g_a:

$$\boxed{g_a = \frac{F_a}{V_a} = \frac{F_a}{d} \cdot \frac{d}{V_a} = \frac{g_{d/V_a}}{g_{d/F_a}}} \tag{4.3}$$

Piezoelectric Sensor Constant Similarly, it is easy to determine the gain from the excitation voltage V_a to the voltage generated by the sensor stack V_s:

$$V_s = g_{V_s/V_a} V_a \tag{4.4}$$

Note here that there is an high pass filter formed by the piezoelectric capacitor and parallel resistor.

The gain can be computed from the dynamical identification and taking the gain at the wanted frequency (above the first resonance).

Using the simscape model, compute the gain at the same frequency from the actuator force F_a to the strain of the sensor stack dl:

$$dl = g_{dl/F_a} F_a \tag{4.5}$$

Then, the "sensor" constant is:

$$\boxed{g_s = \frac{V_s}{dl} = \frac{V_s}{V_a} \cdot \frac{V_a}{F_a} \cdot \frac{F_a}{dl} = \frac{g_{V_s/V_a}}{g_a \cdot g_{dl/F_a}}} \tag{4.6}$$

4.2.2 Identification Data

Let's load the measured FRF from the DAC voltage to the measured encoder and to the sensor stack voltage.

4.2.3 2DoF APA

2DoF APA Let's initialize the APA as a 2DoF model with unity sensor and actuator gains.

Identification without actuator or sensor constants The transfer function from V_a to V_s, d_e and d_a is identified.

Actuator Constant Then, the actuator constant can be computed as shown in Eq. (4.3) by dividing the measured DC gain of the transfer function from V_a to d_e by the estimated DC gain of the transfer function from V_a (in truth the actuator force called F_a) to d_e using the Simscape model.

Results
```
ga = -32.2 [N/V]
```

Sensor Constant Similarly, the sensor constant can be estimated using Eq. (4.6).

Results
```
gs = 0.088 [V/m]
```

Comparison Let's now initialize the APA with identified sensor and actuator constant: And identify the dynamics with included constants. The transfer functions from V_a to d_e are compared in Figure 4.4 and the one from V_a to V_s are compared in Figure 4.5.

figs/apa_act_constant_comp.pdf

Figure 4.4: Comparison of the experimental data and Simscape model (V_a to d_e)

figs/apa_sens_constant_comp.pdf

Figure 4.5: Comparison of the experimental data and Simscape model (V_a to V_s)

Important

The "actuator constant" and "sensor constant" can indeed be identified using this test bench. After identifying these constants, the 2DoF model shows good agreement with the measured dynamics.

figs/apa_comp_model_frf.pdf

4.2.4 Flexible APA

In this section, the sensor and actuator "constants" are also estimated for the flexible model of the APA.

Flexible APA The Simscape APA model is initialized as a flexible one with unity "constants".

Identification without actuator or sensor constants The dynamics from V_a to V_s, d_e and d_a is identified.

Actuator Constant Then, the actuator constant can be computed as shown in Eq. (4.3):

```
Results
ga = 23.5 [N/V]
```

```
Results
gs = -4839841.756 [V/m]
```

Sensor Constant

Comparison Let's now initialize the flexible APA with identified sensor and actuator constant: And identify the dynamics with included constants. The obtained dynamics is compared with the measured one in Figures 4.6 and 4.7.

figs/apa_act_constant_comp_flex.pdf

Figure 4.6: Comparison of the experimental data and Simscape model (u to $d\mathcal{L}_m$)

Important

The flexible model is a bit "soft" as compared with the experimental results.

figs/apa_sens_constant_comp_flex.pdf

Figure 4.7: Comparison of the experimental data and Simscape model (u to τ_m)

4.3 Optimize 2-DoF model to fit the experimental Data

The parameters of the 2DoF model presented in Section 1.1 are now optimize such that the model best matches the measured FRF.

After optimization, the following parameters are used: The dynamics is identified using the Simscape model and compared with the measured FRF in Figure 4.8.

figs/comp_apa_plant_after_opt.pdf

Figure 4.8: Comparison of the measured FRF and the optimized model

Important

The tuned 2DoF is very well representing the (axial) dynamics of the APA.

5 Conclusion