Feedback Control

Dehaeze Thomas[*], Verma Mohit, Zhao Guoying and Collette
Christophe
Precision Mechatronics Laboratory, ULB
Brussels, Belgium
Email: [*]dehaeze.thomas@gmail.com

Abstract—Abstract text to be done

Index Terms—complementary filters, h-infinity, feedback control

I. INTRODUCTION

Model based control
SISO control design methods
- frequency domain techniques
- manual loop-shaping - key idea: modification of the controller
  such that the open-loop is made according to specifications [1].

This works well because the open loop transfer function is linearly
dependent of the controller.

However, the specifications are given in terms of the final system
performance, i.e. as closed-loop specifications.

Norm-based control $\mathcal{H}_\infty$ loop-shaping [2]. Far from standard in
industry as it requires lot of efforts.

Problem of robustness to plant uncertainty:

- Trade off performance / robustness. Difficult to obtain high
  performance in presence of high uncertainty.
- Robust control μ-synthesis. Takes a lot of effort to model the
  plant uncertainty.
- Sensor fusion: combines two sensors using complementary
  filters. The high frequency sensor is collocated with the actuator
  in order to ensure the stability of the system even in presence
  of uncertainty. [3] [4]

Complementary filters: [5].

In this paper, we propose a new controller synthesis method
- based on the use of complementary high pass and low pass filters
- inverse based control
- direct translation of requirements such as disturbance rejection
  and robustness to plant uncertainty

II. THEORY

A. Control Architecture

Let's consider the control architecture represented in Fig. 1 where
G' is the physical plant to control, G is a model of the plant, k is a
gain, H_L and H_H are complementary filters ($H_L + H_H = 1$ in the
complex sense). The signals are the reference signal r, the output
perturbation d_y, the measurement noise n and the control input u.

The dynamics of the closed-loop system is described by the
following equations

$$y = \frac{1 + kGH_H}{1 + L}dy + \frac{kG'}{1 + L}r - \frac{kG'H_L}{1 + L}n \quad (1)$$

$$u = -\frac{kH_L}{1 + L}dy \quad + \frac{k}{1 + L}r - \frac{kH_L}{1 + L}n \quad (2)$$

with $L = k(GH_H + G'H_L)$.

The idea of using such architecture comes from sensor fusion [4],
[3] where we use two sensors. One is measuring the quantity that is
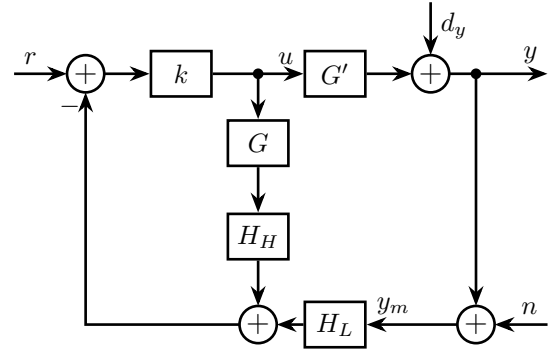required to control, the other is collocated with the actuator in such

Figure 1. Sensor Fusion Architecture

a way that stability is guaranteed. The first one is low pass filtered
in order to obtain good performance at low frequencies and the
second one is high pass filtered to benefits from its good dynamical
properties.

Here, the second sensor is replaced by a model G of the plant
which is assumed to be stable and minimum phase.

One may think that the control architecture shown in Fig. 1
is a multi-loop system, but because no non-linear saturation-type
element is present in the inner-loop (containing k, G and H_H which
are all numerically implemented), the structure is equivalent to the
architecture shown in Fig. 2.
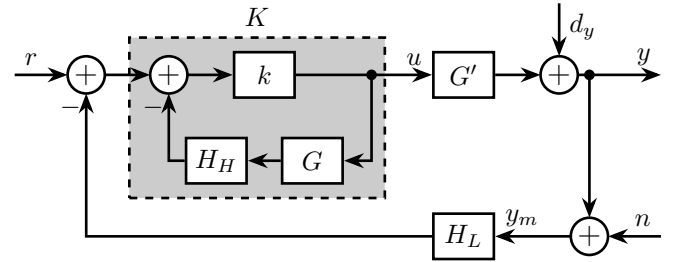

Figure 2. Equivalent feedback architecture

The dynamics of the system can be rewritten as follow

$$y = \frac{1}{1 + G'KH_L}dy + \frac{G'K}{1 + G'KH_L}r - \frac{G'KH_L}{1 + G'KH_L}n \quad (3)$$

$$u = \frac{-KH_L}{1 + G'KH_L}dy + \frac{K}{1 + G'KH_L}r - \frac{KH_L}{1 + G'KH_L}n \quad (4)$$

with $K = \frac{k}{1 + H_H Gk}$

B. Asymptotic behavior

We now want to study the asymptotic system obtained when using
very high value of k

$$\lim_{k \to \infty} K = \lim_{k \to \infty} \frac{k}{1 + H_H Gk} = (H_H G)^{-1} \quad (5)$$

If the obtained K is improper, a low pass filter can be added to have
its causal realization.

Also, we want K to be stable, so G and H_H must be minimum
phase transfer functions.

For now on, we will consider the resulting control architecture
as shown on Fig. 3 where the only "tuning parameters" are the
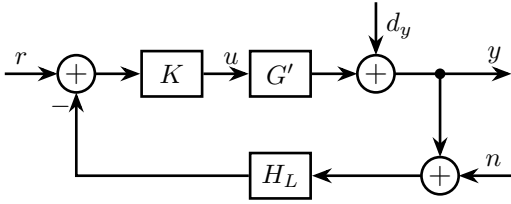complementary filters.

Figure 3. Equivalent classical feedback control architecture

The equations describing the dynamics of the closed-loop system are

$$y = \frac{H_H dy + G'G^{-1}r - G'G^{-1}H_L n}{H_H + G'G^{-1}H_L} \tag{6}$$

$$u = \frac{-G^{-1}H_L dy + G^{-1}r - G^{-1}H_L n}{H_H + G'G^{-1}H_L} \tag{7}$$

At frequencies where the model is accurate: $G^{-1}G' \approx 1$, $H_H + G'G^{-1}H_L \approx H_H + H_L = 1$ and

$$y = H_H dy + r - H_L n \tag{8}$$

$$u = -G^{-1}H_L dy + G^{-1}r - G^{-1}H_L n \tag{9}$$

We obtain a sensitivity transfer function equals to the high pass filter $S = \frac{y}{dy} = H_H$ and a transmissibility transfer function equals to the low pass filter $T = \frac{y}{n} = H_L$.

Assuming that we have a good model of the plant, we have then that the closed-loop behavior of the system converges to the designed complementary filters.

C. Translating the performance requirements into the shapes of the complementary filters

The required performance specifications in a feedback system can usually be translated into requirements on the upper bounds of $|S(j\omega)|$ and $|T(j\omega)|$ [6]. The process of designing a controller $K(s)$ in order to obtain the desired shapes of $|S(j\omega)|$ and $|T(j\omega)|$ is called loop shaping.

The equations (6) and (7) describing the dynamics of the studied feedback architecture are not written in terms of K but in terms of the complementary filters H_L and H_H.

In this section, we then translate the typical specifications into the desired shapes of the complementary filters H_L and H_H.

1) Nominal Stability (NS): The closed-loop system is stable if all its elements are stable (K, G' and H_L) and if the sensitivity function ($S = \frac{1}{1+G'KH_L}$) is stable.

For the nominal system ($G' = G$), we have $S = H_H$.

Nominal stability is then guaranteed if H_L, H_H and G are stable and if G and H_H are minimum phase (to have K stable).

Thus we must design stable and minimum phase complementary filters.

2) Nominal Performance (NP): Typical performance specifications can usually be translated into upper bounds on $|S(j\omega)|$ and $|T(j\omega)|$.

Two performance weights w_H and w_L are defined in such a way that performance specifications are satisfied if

$$|w_H(j\omega)S(j\omega)| \leq 1, \ |w_L(j\omega)T(j\omega)| \leq 1 \quad \forall \omega \tag{10}$$

For the nominal system, we have $S = H_H$ and $T = H_L$, and then nominal performance is ensured by requiring

$$\text{NP} \Leftrightarrow \begin{cases} |w_H(j\omega)H_H(j\omega)| \leq 1 & \forall \omega \quad (11a) \\ |w_L(j\omega)H_L(j\omega)| \leq 1 & \forall \omega \quad (11b) \end{cases}$$

The translation of typical performance requirements on the shapes of the complementary filters is discussed below:
- for disturbance rejections, make $|S| = |H_H|$ small
- for noise attenuation, make $|T| = |H_L|$ small
- for control energy reduction, make $|KS| = |G^{-1}|$ small

We may have other requirements in terms of stability margins, maximum or minimum closed-loop bandwidth.

Closed-Loop Bandwidth: The closed-loop bandwidth ω_B can be defined as the frequency where $|S(j\omega)|$ first crosses $\frac{1}{\sqrt{2}}$ from below.

If one wants the closed-loop bandwidth to be at least ω_B^* (e.g. to stabilize an unstable pole), one can required that $|S(j\omega)| \leq \frac{1}{\sqrt{2}}$ below ω_B^* by designing w_H such that $|w_H(j\omega)| \geq \sqrt{2}$ for $\omega \leq \omega_B^*$.

Similarly, if one wants the closed-loop bandwidth to be less than ω_B^*, one can approximately require that the magnitude of T is less than $\frac{1}{\sqrt{2}}$ at frequencies above ω_B^* by designing w_L such that $|w_L(j\omega)| \geq \sqrt{2}$ for $\omega \geq \omega_B^*$.

Classical stability margins: Gain margin (GM) and phase margin (PM) are usual specifications on controlled system. Minimum GM and PM can be guaranteed by limiting the maximum magnitude of the sensibility function $M_S = \max_\omega |S(j\omega)|$:

$$\text{GM} \geq \frac{M_S}{M_S - 1}; \quad \text{PM} \geq \frac{1}{M_S} \tag{12}$$

Thus, having $M_S \leq 2$ guarantees a gain margin of at least 2 and a phase margin of at least $29°$.

For the nominal system $M_S = \max_\omega |S| = \max_\omega |H_H|$, so one can design w_H so that $|w_H(j\omega)| \geq 1/2$ in order to have

$$|H_H(j\omega)| \leq 2 \quad \forall \omega \tag{13}$$

and thus obtain acceptable stability margins.

Response time to change of reference signal: For the nominal system, the model is accurate and the transfer function from reference signal r to output y is 1 (8) and does not depends of the complementary filters.

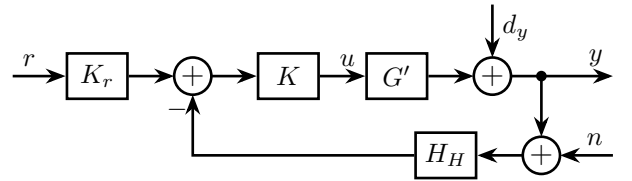However, one can add a pre-filter as shown in Fig. 4.

Figure 4. Prefilter used to limit input usage

The transfer function from y to r becomes $\frac{y}{r} = K_r$ and K_r can we chosen to obtain acceptable response to change of the reference signal. Typically, K_r is a low pass filter of the form

$$K_r(s) = \frac{1}{1 + \tau s} \tag{14}$$

with τ corresponding to the desired response time.

Input usage: Input usage due to disturbances d_y and measurement noise n is determined by $\left|\frac{u}{d_y}\right| = \left|\frac{u}{n}\right| = |G^{-1}H_L|$. Thus it can be limited by setting an upper bound on $|H_L|$.

Input usage due to reference signal r is determined by $\left|\frac{u}{r}\right| = |G^{-1}K_r|$ when using a pre-filter (Fig. 4) and $\left|\frac{u}{r}\right| = |G^{-1}|$ otherwise.

Proper choice of $|K_r|$ is then useful to limit input usage due to change of reference signal.

3) Robust Stability (RS): Robustness stability represents the ability of the control system to remain stable even though there are differences between the actual system G' and the model G that was used to design the controller. These differences can have various origins such as unmodelled dynamics or non-linearities.

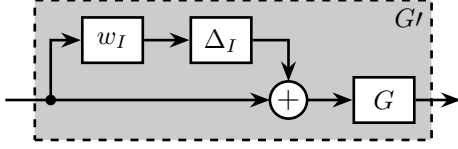To represent the differences between the model and the actual system, one can choose to use the general input multiplicative uncertainty as represented in Fig. 5.

Figure 5. Input multiplicative uncertainty

Then, the set of possible perturbed plant is described by

$$\Pi_i : \quad G_p(s) = G(s)\big(1 + w_I(s)\Delta_I(s)\big); \quad |\Delta_I(j\omega)| \le 1 \ \forall\omega \quad (15)$$

and w_I should be chosen such that all possible plants G' are contained in the set Π_i.

Using input multiplicative uncertainty, robust stability is equivalent to have [2]:

$$\begin{aligned}
\text{RS} &\Leftrightarrow |w_I T| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega \\
&\Leftrightarrow \left| w_I \frac{G'KH_L}{1 + G'KH_L} \right| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega \\
&\Leftrightarrow \left| w_I \frac{G'G^{-1}H_H{}^{-1}H_L}{1 + G'G^{-1}H_H{}^{-1}H_L} \right| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega \\
&\Leftrightarrow \left| w_I \frac{(1 + w_I\Delta)H_H{}^{-1}H_L}{1 + (1 + w_I\Delta)H_H{}^{-1}H_L} \right| \le 1 \quad \forall\Delta, \ |\Delta| \le 1, \ \forall\omega \\
&\Leftrightarrow \left| w_I \frac{(1 + w_I\Delta)H_L}{1 + w_I\Delta H_L} \right| \le 1 \quad \forall\Delta, \ |\Delta| \le 1, \ \forall\omega \\
&\Leftrightarrow |H_L w_I| \frac{1 + |w_I|}{1 - |w_I H_L|} \le 1, \quad 1 - |w_I H_L| > 0 \quad \forall\omega \\
&\Leftrightarrow |H_L w_I|(2 + |w_I|) \le 1, \quad 1 - |w_I H_L| > 0 \quad \forall\omega \\
&\Leftrightarrow |H_L w_I|(2 + |w_I|) \le 1 \quad \forall\omega
\end{aligned}$$

Robust stability is then guaranteed by having the low pass filter H_L satisfying (16).

$$\text{RS} \Leftrightarrow |H_L| \le \frac{1}{|w_I|(2 + |w_I|)} \quad \forall\omega \quad (16)$$

To ensure robust stability condition (11b) can be used if w_L is designed in such a way that $|w_L| \ge |w_I|(2 + |w_I|)$.

4) Robust Performance (RP): Robust performance is a property for a controlled system to have its performance guaranteed even though the dynamics of the plant is changing within specified bounds.

For robust performance, we then require to have the performance condition valid for all possible plants in the defined uncertainty set:

$$\text{RP} \Leftrightarrow \begin{cases} |w_H S| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega & (17a) \\ |w_L T| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega & (17b) \end{cases}$$

Let's transform condition (17a) into a condition on the complementary filters

$$\begin{aligned}
&|w_H S| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega \\
\Leftrightarrow & \left| w_H \frac{1}{1 + G'G^{-1}H_H^{-1}H_L} \right| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega \\
\Leftrightarrow & \left| \frac{w_H H_H}{1 + \Delta w_I H_L} \right| \le 1 \quad \forall\Delta, \ |\Delta| \le 1, \ \forall\omega \\
\Leftrightarrow & \frac{|w_H H_H|}{1 - |w_I H_L|} \le 1, \ \forall\omega \\
\Leftrightarrow & |w_H H_H| + |w_I H_L| \le 1, \ \forall\omega
\end{aligned}$$

The same can be done with condition (17b)

$$\begin{aligned}
&|w_L T| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega \\
\Leftrightarrow & \left| w_L \frac{G'G^{-1}H_H^{-1}H_L}{1 + G'G^{-1}H_H^{-1}H_L} \right| \le 1 \quad \forall G' \in \Pi_I, \ \forall\omega \\
\Leftrightarrow & \left| w_L H_L \frac{1 + w_I \Delta}{1 + w_I \Delta H_L} \right| \le 1 \quad \forall\Delta, \ |\Delta| \le 1, \ \forall\omega \\
\Leftrightarrow & |w_L H_L| \frac{1 + |w_I|}{1 - |w_I H_L|} \le 1 \quad \forall\omega \\
\Leftrightarrow & |H_L| \le \frac{1}{|w_L|(1 + |w_I|) + |w_I|} \quad \forall\omega
\end{aligned}$$

Robust performance is then guaranteed if (18a) and (18b) are satisfied.

$$\text{RP} \Leftrightarrow \begin{cases} |w_H H_H| + |w_I H_L| \le 1, \ \forall\omega & (18a) \\ |H_L| \le \dfrac{1}{|w_L|(1 + |w_I|) + |w_I|} \quad \forall\omega & (18b) \end{cases}$$

One should be aware than when looking for a robust performance condition, only the worst case is evaluated and using the robust stability condition may lead to conservative control.

D. Procedure

In order to apply this control technique, we propose the following procedure:

1) Identify the plant to be controlled in order to obtain G
2) Design the weighting function w_I such that all possible plants G' are contained in the set Π_i
3) Translate the performance requirements into upper bounds on the complementary filters (as explained in Sec. II-C)
4) Design the weighting functions w_H and w_L and generate the complementary filters using $\mathcal{H}_\infty$-synthesis (as further explained in Sec. III). If the synthesis fails to give filters satisfying the upper bounds previously defined, either the requirements have to be reworked or a better model G that will permits to have a smaller w_I should be obtained. If one does not want to use the $\mathcal{H}_\infty$ synthesis, one can use pre-made complementary filters given in Sec. III-E
5) If $K = (GH_H)^{-1}$ is not proper, a low pass filter should be added
6) Design a pre-filter K_r if requirements on input usage or response to reference change are not met
7) Control implementation: Filter the measurement with H_L, implement the controller K and the pre-filter K_r as shown on Fig. 4

This procedure will be applied on a numerical model in Sec. IV.

III. H-INFINITY SYNTHESIS OF COMPLEMENTARY FILTERS

First order complementary filters are easy to synthesize. For instance, one can use the following filters

$$H_H(s) = \frac{s/\omega_0}{1 + s/\omega_0}; \quad H_L(s) = \frac{1}{1 + s/\omega_0} \qquad (19)$$

with ω_0 is the tuning parameter corresponding to the crossover frequency of the filters.

However, the manual design of higher order complementary filters is far more complex and we have to use an automatic synthesis technique.

As shown in Sec. II-C, most of the performance requirements can be expressed as upper bounds on the magnitude of the complementary filters.

Thus, the $\mathcal{H}_\infty$ framework seems adapted and we here propose a technique to synthesis complementary filters while specifying uppers bounds on their magnitudes.

A. $\mathcal{H}_\infty$ problem formulation

In this section, we formulate the $\mathcal{H}_\infty$ problem for the synthesis of complementary filters.

The synthesis objective is to shape an high pass filter H_H and a low pass filter H_L while ensuring their complementary property ($H_H + H_L = 1$).

To do so, we define two weighting functions w_L and w_H that will respectively used to shape H_L and H_H.

The synthesis problem is then

$$\text{Find } H_L, H_H \text{ such that} \begin{cases} H_L \text{ and } H_H \text{ are stable} & (20a) \\ H_L + H_H = 1 & (20b) \\ |w_L H_L| \le 1 \quad \forall \omega & (20c) \\ |w_H H_H| \le 1 \quad \forall \omega & (20d) \end{cases}$$

To express this synthesis problem into an $\mathcal{H}_\infty$ synthesis problem, we define the following generalized plant P (also shown on Fig. 6):

$$\begin{bmatrix} w \\ u \end{bmatrix} = P \begin{bmatrix} z_H \\ z_L \\ v \end{bmatrix}; \quad P = \begin{bmatrix} w_H & -w_H \\ 0 & w_L \\ 1 & 0 \end{bmatrix} \qquad (21)$$
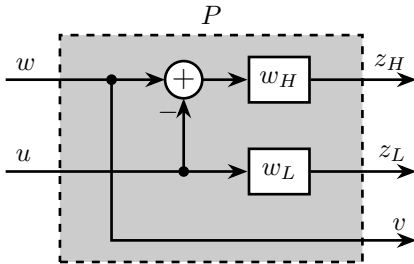

Figure 6.   Generalized plant for the synthesis of the complementary filters

The $\mathcal{H}_\infty$ synthesis objective is then to design a stable filter H_L (Fig. 7) such that the $\mathcal{H}_\infty$ norm of the transfer function from w to $[z_H, \ z_L]$ is less than 1:

$$\left\| \begin{matrix} (1 - H_L)w_H \\ H_L w_L \end{matrix} \right\|_\infty \le 1 \qquad (22)$$

Which is equivalent to

$$\left\| \begin{matrix} H_H w_H \\ H_L w_L \end{matrix} \right\|_\infty < 1 \text{ by choosing } H_H = 1 - H_L \qquad (23)$$

Performance conditions (20c) and (20c) are satisfied by (23). Complementary condition (20b) is satisfied by design: $H_H = 1 - H_L$
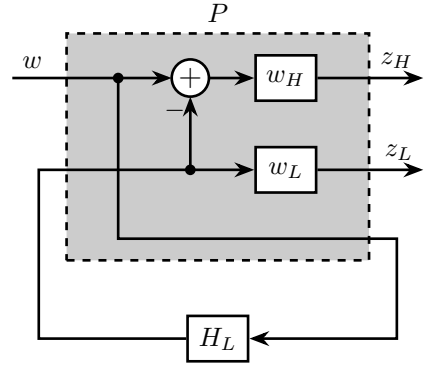

Figure 7. $\mathcal{H}_\infty$-synthesis of complementary filters

and thus $H_L + H_H = 1$. The stability condition (20a) is guaranteed by the H_∞ synthesis (**reference**).

Using this synthesis method, we are then able to shape at the same time the high pass and low pass filters while ensuring their complementary.

B. Control requirements as $\mathcal{H}_\infty$ norm of complementary filters

As presented in Sec. II-C, almost all the requirements can be specified with upper bounds on the complementary filters. However, robust performance condition (18a) is not.

With the $\mathcal{H}_\infty$ synthesis the condition (23) only ensure

$$\left\| \begin{matrix} H_H w_H \\ H_L w_L \end{matrix} \right\|_\infty \le 1 \Leftrightarrow \max_\omega \sqrt{|w_L H_L|^2 + |w_H H_H|^2} \le 1$$

$$\Rightarrow |w_L H_L| + |w_H H_H| \le \sqrt{2} \quad \forall \omega$$

And thus we have almost robust stability.

C. Choice of the weighting functions

We here give some advice on the choice of the weighting functions used for the synthesis of the complementary filters.

The shape should be such that the performance requirements are met as explain in Sec. II-C.

However, one should be careful when designing the complementary filters, and should only use stable and minimum phase transfer functions. The order of the weights should stay reasonably small as this will increase the complexity of the optimization problem.

One should not forget the fundamental limitations of feedback control such that $S + T = 1$. Similarly, we here have that $H_L + H_H = 1$ which implies that H_L and H_H cannot be made small at the same time.

D. Trade-off between performance and robustness

E. Analytical formula of complementary filters

To simplify the synthesis, one can use already synthesized filters

$$H_L(s) = \frac{1}{1 + \frac{s}{\omega_0}} \qquad (24)$$

$$H_H(s) = \frac{\frac{s}{\omega_0}}{1 + \frac{s}{\omega_0}} \qquad (25)$$

$$H_L(s) = \frac{(1 + \alpha)(\frac{s}{\omega_0}) + 1}{\left((\frac{s}{\omega_0}) + 1 \right) \left((\frac{s}{\omega_0})^2 + \alpha(\frac{s}{\omega_0}) + 1 \right)} \qquad (26)$$

$$H_H(s) = \frac{(\frac{s}{\omega_0})^2 \left((\frac{s}{\omega_0}) + 1 + \alpha \right)}{\left((\frac{s}{\omega_0}) + 1 \right) \left((\frac{s}{\omega_0})^2 + \alpha(\frac{s}{\omega_0}) + 1 \right)} \qquad (27)$$

$$H_L(s) = \frac{(1 + (\alpha+1)(\beta+1))\left(\frac{s}{\omega_0}\right)^2 + (1 + \alpha + \beta)(\frac{s}{\omega_0}) + 1}{\left(\frac{s}{\omega_0} + 1\right)\left((\frac{s}{\omega_0})^2 + \alpha(\frac{s}{\omega_0}) + 1\right)\left((\frac{s}{\omega_0})^2 + \beta(\frac{s}{\omega_0}) + 1\right)} \tag{28}$$

$$H_H(s) = \frac{\left(\frac{s}{\omega_0}\right)^3 \left((\frac{s}{\omega_0})^2 + (1 + \alpha + \beta)(\frac{s}{\omega_0}) + (1 + (\alpha+1)(\beta+1))\right)}{\left(\frac{s}{\omega_0} + 1\right)\left((\frac{s}{\omega_0})^2 + \alpha(\frac{s}{\omega_0}) + 1\right)\left((\frac{s}{\omega_0})^2 + \beta(\frac{s}{\omega_0}) + 1\right)} \tag{29}$$

IV. NUMERICAL SIMULATIONS

In this section, the **xxx** method is applied on a simple control problem.

A. Plant

Let's consider the problem of controlling an active vibration isolation system that consist of a mass m to be isolated, a piezoelectric actuator and a geophone.

We represent this system by a mass-spring-damper system as shown Fig. 8 where m typically represents the mass of the payload to be isolated, k and c represent respectively the stiffness and damping of the mount. w is the ground motion. The values for the parameters of the models are

$$m = 20\,\text{N/m}; \quad k = 10^4 \text{N/m}; \quad c = 10^2 \text{N/(m/s)}$$
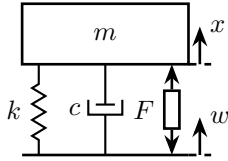
Figure 8. Model of the positioning system

The model of the plant $G(s)$ from actuator force F to displacement x is then

$$G(s) = \frac{1}{ms^2 + cs + k} \tag{30}$$
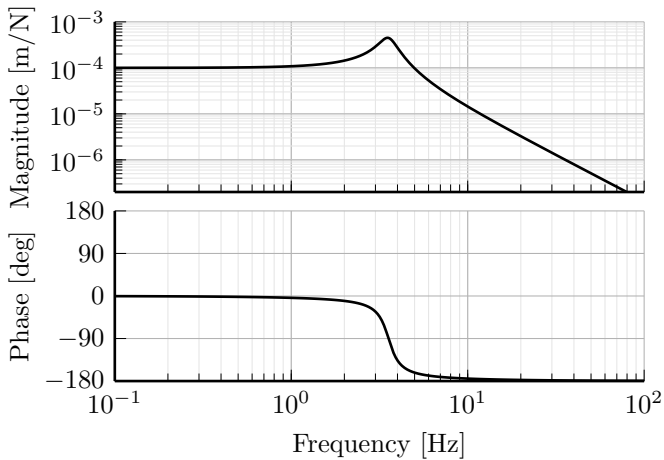
Its bode plot is shown on Fig. 9.

Figure 9. Bode plot of the transfer function $G(s)$ from F to x

B. Requirements

The control objective is to isolate the displacement x of the mass from the ground motion w.

The disturbance rejection should be at least 10 at 2 Hz and with a slope of -2 below 2 Hz until a rejection of 10^4.

Closed-loop bandwidth should be less than 20 Hz (because of time delay induced by limited sampling frequency?).

Noise attenuation should be at least 10 above 40 Hz and 100 above 500 Hz

Robustness to unmodelled dynamics. We model the uncertainty on the dynamics of the plant by a multiplicative weight

$$w_I(s) = \frac{\tau s + r_0}{(\tau/r_\infty)s + 1} \tag{31}$$

where $r_0 = 0.1$ is the relative uncertainty at steady-state, $1/\tau = 100$ Hz is the frequency at which the relative uncertainty reaches 100%, and $r_\infty = 10$ is the magnitude of the weight at high frequency.

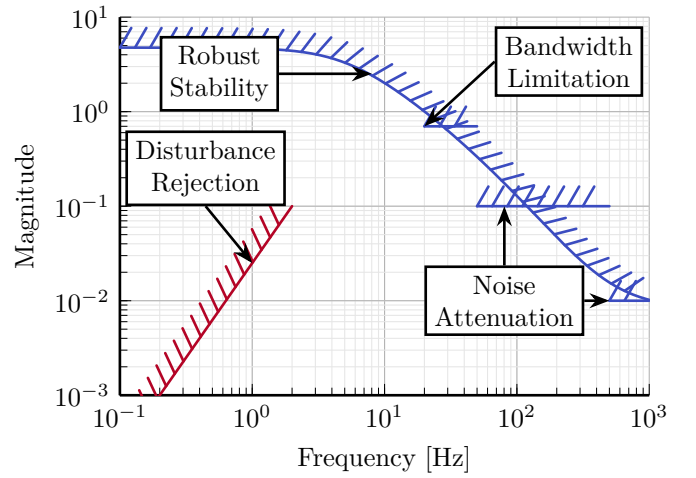All the requirements on H_L and H_H are represented on Fig. 10.

Figure 10. Specifications

C. Design of the filters

We then design w_L and w_H such that their magnitude are below the upper bounds shown on Fig. 11.

$$w_L = \frac{(s + 22.36)^2}{0.005(s + 1000)^2} \tag{32a}$$

$$w_H = \frac{1}{0.0005(s + 0.4472)^2} \tag{32b}$$

After the $\mathcal{H}_\infty$-synthesis, we obtain H_L and H_H, and we plot their magnitude on phase on Fig. 11.

$$H_L = \frac{0.0063957(s + 1016)(s + 985.4)(s + 26.99)}{(s + 57.99)(s^2 + 65.77s + 2981)} \tag{33a}$$

$$H_H = \frac{0.9936(s + 111.1)(s^2 + 0.3988s + 0.08464)}{(s + 57.99)(s^2 + 65.77s + 2981)} \tag{33b}$$

*1) **TODO** Plot the complementary filters with the phase?:*

D. Controller analysis

The controller is $K = (H_H G)^{-1}$. A low pass filter is added to K so that it is proper and implementable.

The obtained controller is shown on Fig. 12.

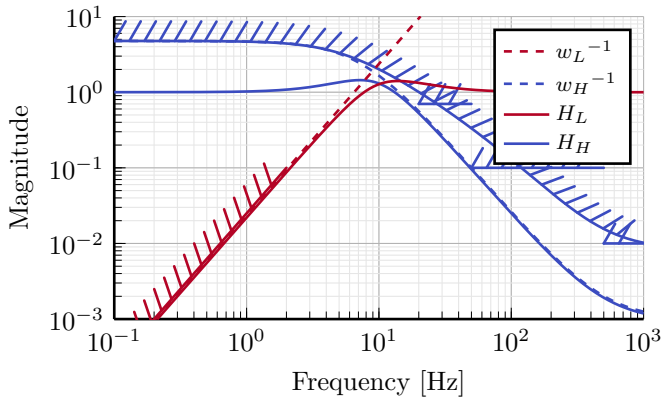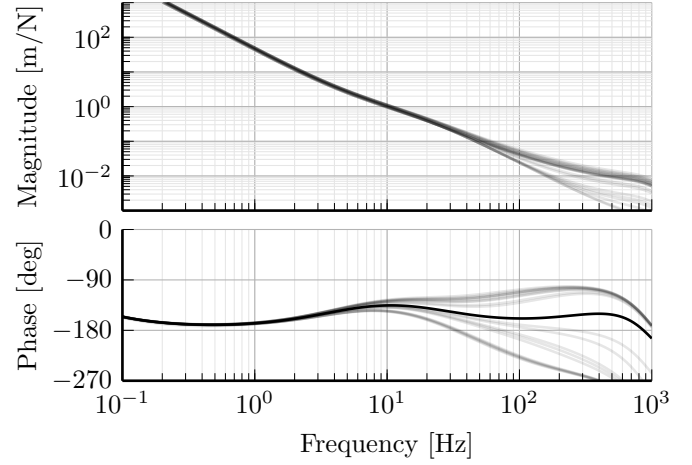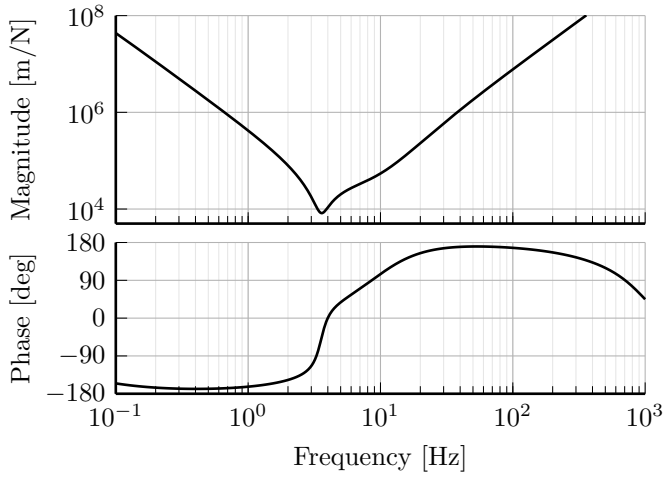It is implemented as shown on Fig. 13.

The loop gain is

Figure 11. Bode plot of the obtained complementary filters

Figure 12. Bode plot of the controller K

E. Robustness analysis

The robust stability can be access on the nyquist plot (Fig. 15). The robust performance is shown on Fig. 16.
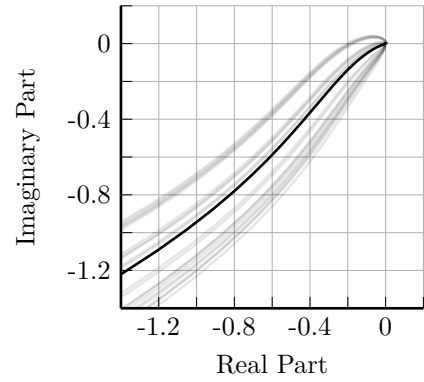
V. DISCUSSION

A. Limitations

One should be aware that this control architecture suffers from the same limitations as classical feedback control. $S + T = 1$ similarly to $H_H + H_L = 1$.

Should I put that at the end of the paper?

- The synthesis of the filters does not take into account the plant. The limitation linked to the filters are fundamental limitations of feedback control: $S + T = 1$.
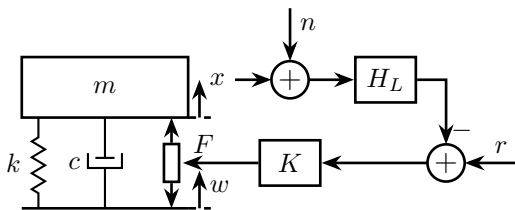

Figure 13. Control of a positioning system

Figure 14. Bode plot of the loop gain KGH_L

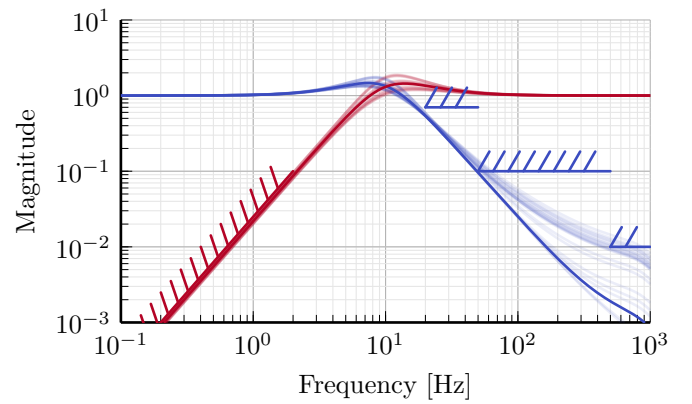Figure 15. Nyquist plot of the uncertain system

Figure 16. Robust Performance

- The limitations are appearing when inverting the model of the plant (inverse based control) => RHP-zeros are becoming RHP-poles

VI. Conclusion

In the following work, this will be generalized to MIMO systems

VII. Acknowledgment

References

[1] T. Oomen, "Advanced motion control for precision mechatronics: Control, identification, and learning of complex systems," *IEEJ Journal of Industry Applications*, vol. 7, no. 2, pp. 127–140, 2018. [Online]. Available: https://doi.org/10.1541/ieejjia.7.127

[2] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*. John Wiley, 2007.

[3] C. Collette and F. Matichard, "Sensor fusion methods for high performance active vibration isolation systems," *Journal of Sound and Vibration*, vol. 342, no. nil, pp. 1–21, 2015. [Online]. Available: https://doi.org/10.1016/j.jsv.2015.01.006

[4] ——, "Vibration control of flexible structures using fusion of inertial sensors and hyper-stable actuator-sensor pairs," in *International Conference on Noise and Vibration Engineering (ISMA2014)*, 2014.

[5] W. Hua, "Low frequency vibration isolation and alignment system for advanced ligo," Ph.D. dissertation, stanford university, 2005.

[6] J. E. Bibel and D. S. Malyevac, "Guidelines for the selection of weighting functions for h-infinity control," NAVAL SURFACE WARFARE CENTER DAHLGREN DIV VA, Tech. Rep., 1992.