Optimal and Robust Sensor Fusion

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***Abstract*—Abstract text to be done**

***Index Terms*—Complementary Filters, Sensor Fusion, H-Infinity Synthesis**

I. Introduction

Section II Section III Section IV Section V

II. Optimal Super Sensor Noise: $\mathcal{H}_2$ Synthesis

A. Sensor Model

Let's consider a sensor measuring a physical quantity x (Figure 1). The sensor has an internal dynamics which is here modelled with a Linear Time Invariant (LTI) system transfer function $G_i(s)$.

The noise of sensor can be described by the Power Spectral Density (PSD) $\Phi_{n_i}(\omega)$.

This is approximated by shaping a white noise with unitary PSD $\tilde{n}_i$ (2) with a LTI transfer function $N_i(s)$:

$$\begin{aligned}\Phi_{n_i}(\omega) &= \left|N_i(j\omega)\right|^2 \Phi_{\tilde{n}_i}(\omega) \\ &= \left|N_i(j\omega)\right|^2\end{aligned} \tag{1}$$

$$\Phi_{\tilde{n}_i}(\omega) = 1 \tag{2}$$

The output of the sensor v_i:

$$v_i = (G_i)\, x + (G_i N_i)\, \tilde{n}_i \tag{3}$$

In order to obtain an estimate $\hat{x}_i$ of x, a model $\hat{G}_i$ of the (true) sensor dynamics G_i is inverted and applied at the output (Figure 1):

$$\hat{x}_i = \left(\hat{G}_i^{-1} G_i\right) x + \left(\hat{G}_i^{-1} G_i N_i\right) \tilde{n}_i \tag{4}$$

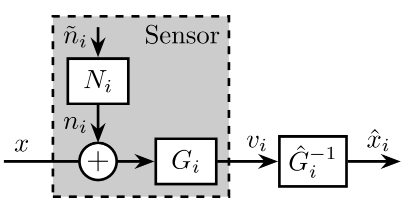

Fig. 1. Sensor Model

B. Sensor Fusion Architecture

Let's now consider two sensors measuring the same physical quantity x but with different dynamics (G_1, G_2) and noise characteristics (N_1, N_2) (Figure 2).

The noise sources $\tilde{n}_1$ and $\tilde{n}_2$ are considered to be uncorrelated.

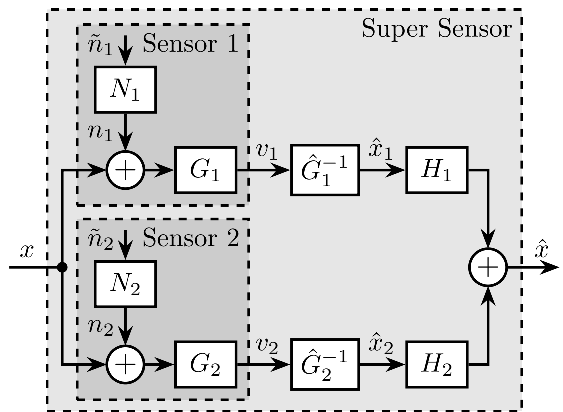

Fig. 2. Sensor Fusion Architecture with sensor noise

The output of both sensors $(v1, v2)$ are then passed through the inverse of the sensor model to obtained two estimates $(\hat{x}_1, \hat{x}_2)$ of x. These two estimates are then filtered out by two filters H_1 and H_2 and summed to gives the super sensor estimate $\hat{x}$.

$$\begin{aligned}\hat{x} = &\left(H_1 \hat{G}_1^{-1} G_1 + H_2 \hat{G}_2^{-1} G_2\right) x \\ &+ \left(H_1 \hat{G}_1^{-1} G_1 N_1\right) \tilde{n}_1 + \left(H_2 \hat{G}_2^{-1} G_2 N_2\right) \tilde{n}_2\end{aligned} \tag{5}$$

Suppose the sensor dynamical model $\hat{G}_i$ is perfect:

$$\hat{G}_i = G_i \tag{6}$$

We considered here complementary filters:

$$H_1(s) + H_2(s) = 1 \tag{7}$$

In such case, the super sensor estimate $\hat{x}$ is equal to x plus the noise of the individual sensors filtered out by the complementary filters:

$$\hat{x} = x + (H_1 N_1)\, \tilde{n}_1 + (H_2 N_2)\, \tilde{n}_2 \tag{8}$$

C. Super Sensor Noise

Let's note n the super sensor noise.

$$n = (H_1 N_1)\,\tilde{n}_1 + (H_2 N_2)\,\tilde{n}_2 \tag{9}$$

As the noise of both sensors are considered to be uncorrelated, the PSD of the super sensor noise is computed as follow:

$$\Phi_n(\omega) = |H_1 N_1|^2 + |H_2 N_2|^2 \tag{10}$$

It is clear that the PSD of the super sensor depends on the norm of the complementary filters.

D. $\mathcal{H}_2$ Synthesis of Complementary Filters

The goal is to design $H_1(s)$ and $H_2(s)$ such that the effect of the noise sources $\tilde{n}_1$ and $\tilde{n}_2$ has the smallest possible effect on the noise n of the estimation $\hat{x}$.

And the goal is the minimize the Root Mean Square (RMS) value of n:

$$\sigma_n = \sqrt{\int_0^\infty \Phi_{\hat{n}}(\omega) d\omega} = \left\| \begin{matrix} H_1 N_1 \\ H_2 N_2 \end{matrix} \right\|_2 \tag{11}$$

Thus, the goal is to design $H_1(s)$ and $H_2(s)$ such that $H_1(s) + H_2(s) = 1$ and such that $\left\| \begin{matrix} H_1 N_1 \\ H_2 N_2 \end{matrix} \right\|_2$ is minimized.

$$\begin{pmatrix} z_1 \\ z_2 \\ v \end{pmatrix} = \begin{bmatrix} N_1 & N_1 \\ 0 & N_2 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} w \\ u \end{pmatrix} \tag{12}$$

The $\mathcal{H}_2$ synthesis of the complementary filters thus minimized the RMS value of the super sensor noise.

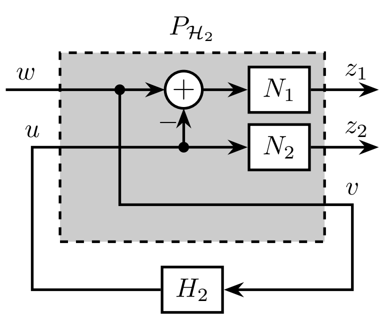

Fig. 3. Generalized plant $P_{\mathcal{H}_2}$ used for the $\mathcal{H}_2$ synthesis of complementary filters

E. Example

F. Robustness Problem

III. Robust Sensor Fusion: $\mathcal{H}_\infty$ Synthesis

A. Representation of Sensor Dynamical Uncertainty

Suppose that the sensor dynamics $G_i(s)$ can be modelled by a nominal d

$$G_i(s) = \hat{G}_i(s)\,(1 + w_i(s)\Delta_i(s))\,; \quad |\Delta_i(j\omega)| < 1 \forall \omega \tag{13}$$

B. Sensor Fusion Architecture

$$\begin{aligned} \hat{x} = \Big(& H_1 \hat{G}_1^{-1} \hat{G}_1 (1 + w_1 \Delta_1) \\ & + H_2 \hat{G}_2^{-1} \hat{G}_2 (1 + w_2 \Delta_2) \Big) x \end{aligned} \tag{14}$$

with Δ_i is any transfer function satisfying $\|\Delta_i\|_\infty < 1$.

Suppose the model inversion is equal to the nominal model:

$$\hat{G}_i = G_i \tag{15}$$

$$\hat{x} = (1 + H_1 w_1 \Delta_1 + H_2 w_2 \Delta_2)\,x \tag{16}$$

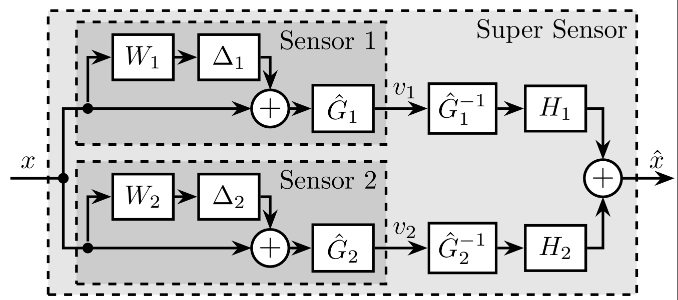

Fig. 4. Sensor Fusion Architecture with sensor model uncertainty

C. Super Sensor Dynamical Uncertainty

The uncertainty set of the transfer function from $\hat{x}$ to x at frequency ω is bounded in the complex plane by a circle centered on 1 and with a radius equal to $|w_1(j\omega)H_1(j\omega)| + |w_2(j\omega)H_2(j\omega)|$.

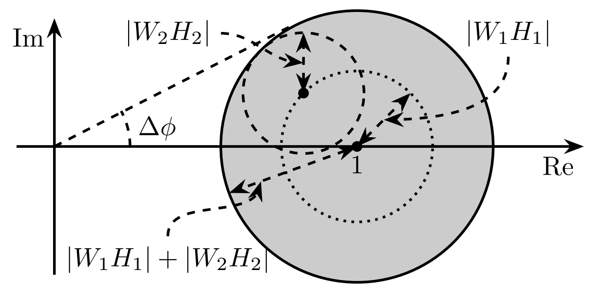

Fig. 5. Super Sensor model uncertainty displayed in the complex plane

D. $\mathcal{H}_\infty$ Synthesis of Complementary Filters

In order to minimize the super sensor dynamical uncertainty

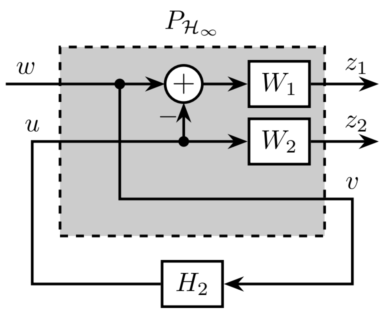

Fig. 6. Generalized plant $P_{\mathcal{H}_\infty}$ used for the $\mathcal{H}_\infty$ synthesis of complementary filters

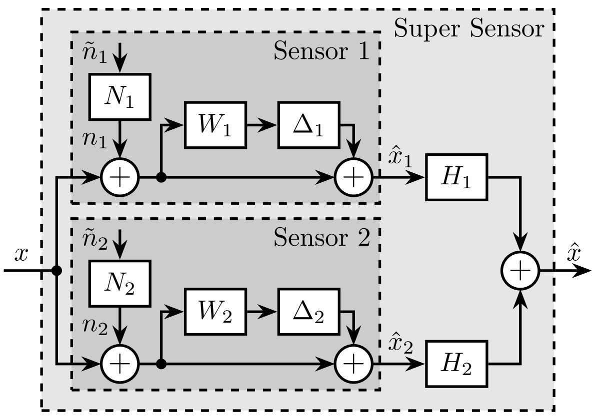

Fig. 7. Super Sensor Fusion with both sensor noise and sensor model uncertainty

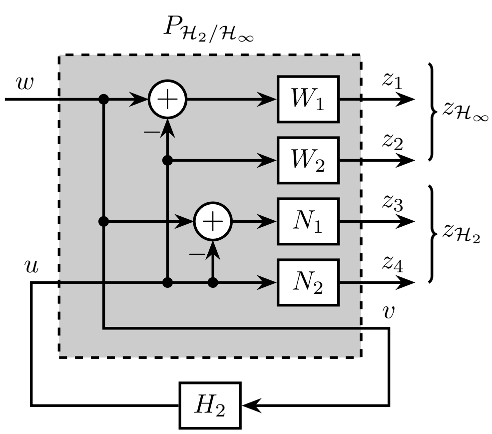

Fig. 8. Generalized plant $P_{\mathcal{H}_2/H_\infty}$ used for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis of complementary filters

E. Example

IV. Optimal and Robust Sensor Fusion: Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

A. Sensor Fusion Architecture

B. Synthesis Objective

C. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

D. Example

V. Experimental Validation

A. Experimental Setup

B. Sensor Noise and Dynamical Uncertainty

C. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

D. Super Sensor Noise and Dynamical Uncertainty

VI. Conclusion

VII. Acknowledgment

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