Robust and Optimal Sensor Fusion

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***Abstract*—Abstract text to be done**

***Index Terms*—Complementary Filters, Sensor Fusion, H-Infinity Synthesis**

I. Introduction

II. Optimal Super Sensor Noise: $\mathcal{H}_2$ Synthesis

A. Sensor Model

B. Sensor Fusion Architecture

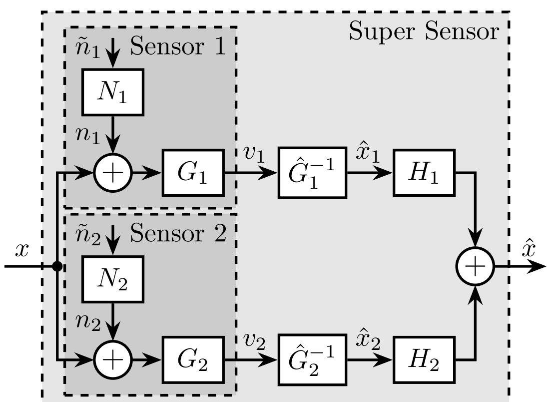

Fig. 1. Sensor Fusion Architecture with sensor noise

Let note Φ the PSD. $\tilde{n}_1$ and $\tilde{n}_2$ are white noise with unitary power spectral density:

$$\Phi_{\tilde{n}_i}(\omega) = 1 \quad (1)$$

$$\begin{aligned}\hat{x} = &\left(H_1 \hat{G}_1^{-1} G_1 + H_2 \hat{G}_2^{-1} G_2\right) x \\ &+ \left(H_1 \hat{G}_1^{-1} N_1\right) \tilde{n}_1 + \left(H_2 \hat{G}_2^{-1} N_2\right) \tilde{n}_2\end{aligned} \quad (2)$$

Suppose the sensor dynamical model $\hat{G}_i$ is perfect:

$$\hat{G}_i = G_i \quad (3)$$

Complementary Filters

$$H_1(s) + H_2(s) = 1 \quad (4)$$

$$\hat{x} = x + \left(H_1 \hat{G}_1^{-1} N_1\right) \tilde{n}_1 + \left(H_2 \hat{G}_2^{-1} N_2\right) \tilde{n}_2 \quad (5)$$

Perfect dynamics + filter noise

C. Super Sensor Noise

Let's note n the super sensor noise.

Its PSD is determined by:

$$\Phi_n(\omega) = \left|H_1 \hat{G}_1^{-1} N_1\right|^2 + \left|H_2 \hat{G}_2^{-1} N_2\right|^2 \quad (6)$$

D. $\mathcal{H}_2$ Synthesis of Complementary Filters

The goal is to design $H_1(s)$ and $H_2(s)$ such that the effect of the noise sources $\tilde{n}_1$ and $\tilde{n}_2$ has the smallest possible effect on the noise n of the estimation $\hat{x}$.

And the goal is the minimize the Root Mean Square (RMS) value of n:

$$\sigma_n = \sqrt{\int_0^\infty \Phi_{\hat{n}}(\omega) d\omega} = \left\| \begin{matrix} \hat{G}_1^{-1} N_1 H_1 \\ \hat{G}_2^{-1} N_2 H_2 \end{matrix} \right\|_2 \quad (7)$$

Thus, the goal is to design $H_1(s)$ and $H_2(s)$ such that $H_1(s)+H_2(s) = 1$ and such that $\left\| \begin{matrix} \hat{G}_1^{-1} N_1 H_1 \\ \hat{G}_2^{-1} N_2 H_2 \end{matrix} \right\|_2$ is minimized.

$$\begin{pmatrix} z_1 \\ z_2 \\ v \end{pmatrix} = \begin{bmatrix} \hat{G}_1^{-1} N_1 & -\hat{G}_1^{-1} N_1 \\ 0 & \hat{G}_2^{-1} N_2 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} w \\ u \end{pmatrix} \quad (8)$$

The $\mathcal{H}_2$ synthesis of the complementary filters thus minimized the RMS value of the super sensor noise.

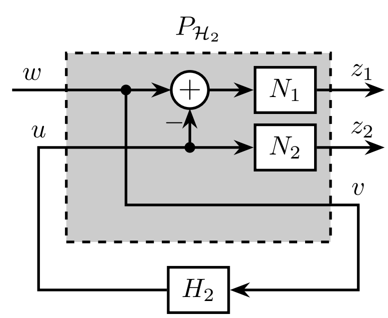

Fig. 2. Generalized plant $P_{\mathcal{H}_2}$ used for the $\mathcal{H}_2$ synthesis of complementary filters

E. Example

F. Robustness Problem

III. Robust Sensor Fusion: $\mathcal{H}_\infty$ Synthesis

A. Representation of Sensor Dynamical Uncertainty

Suppose that the sensor dynamics $G_i(s)$ can be modelled by a nominal d

$$G_i(s) = \hat{G}_i(s)\left(1 + w_i(s)\Delta_i(s)\right); \quad |\Delta_i(j\omega)| < 1 \forall \omega \tag{9}$$

B. Sensor Fusion Architecture

$$\begin{aligned} \hat{x} = \Big(& H_1 \hat{G}_1^{-1} \hat{G}_1 (1 + w_1 \Delta_1) \\ & + H_2 \hat{G}_2^{-1} \hat{G}_2 (1 + w_2 \Delta_2) \Big) x \end{aligned} \tag{10}$$

with Δ_i is any transfer function satisfying $\|\Delta_i\|_\infty < 1$.

Suppose the model inversion is equal to the nominal model:

$$\hat{G}_i = G_i \tag{11}$$

$$\hat{x} = \left(1 + H_1 w_1 \Delta_1 + H_2 w_2 \Delta_2\right) x \tag{12}$$

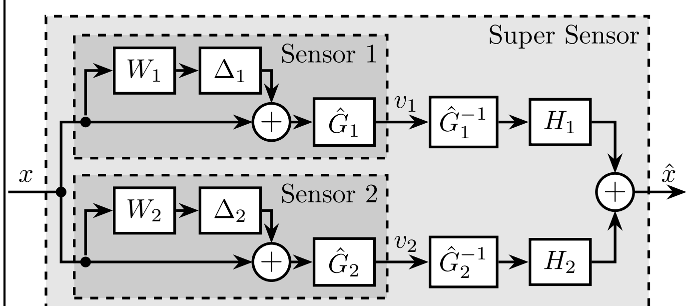

Fig. 3. Sensor Fusion Architecture with sensor model uncertainty

C. Super Sensor Dynamical Uncertainty

The uncertainty set of the transfer function from $\hat{x}$ to x at frequency ω is bounded in the complex plane by a circle centered on 1 and with a radius equal to $|w_1(j\omega)H_1(j\omega)| + |w_2(j\omega)H_2(j\omega)|$.

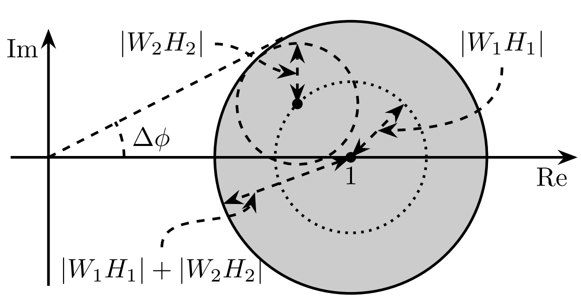

Fig. 4. Super Sensor model uncertainty displayed in the complex plane

D. $\mathcal{H}_\infty$ Synthesis of Complementary Filters

In order to minimize the super sensor dynamical uncertainty

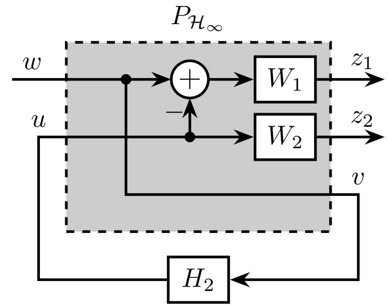

Fig. 5. Generalized plant $P_{\mathcal{H}_\infty}$ used for the $\mathcal{H}_\infty$ synthesis of complementary filters

E. Example

IV. Optimal and Robust Sensor Fusion: Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

A. Sensor Fusion Architecture

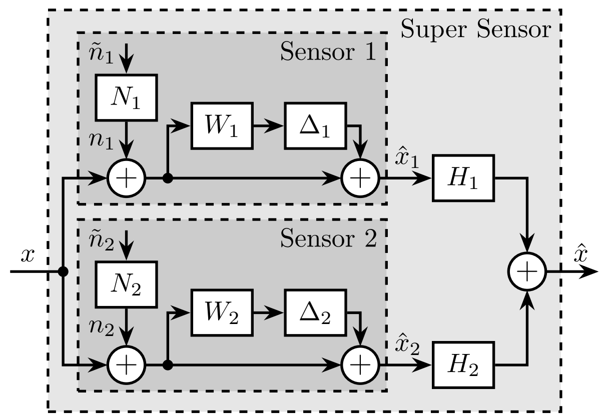

Fig. 6. Super Sensor Fusion with both sensor noise and sensor model uncertainty

B. Synthesis Objective

C. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

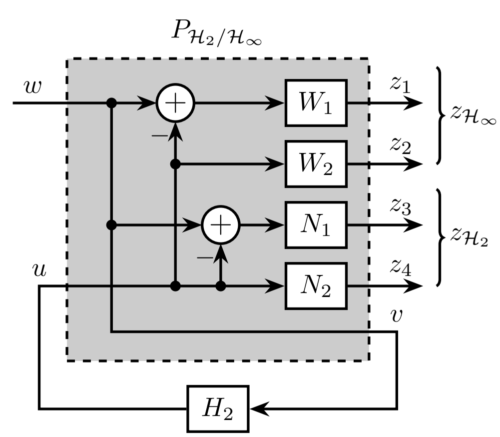

Fig. 7. Generalized plant $P_{\mathcal{H}_2/H_\infty}$ used for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis of complementary filters

D. Example

V. Experimental Validation

A. Experimental Setup

B. Sensor Noise and Dynamical Uncertainty

C. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

D. Super Sensor Noise and Dynamical Uncertainty

VI. Conclusion

VII. Acknowledgment

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