Active Damping of Rotating Positioning Platforms

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Abstract
Abstract text to be done

1 Introduction

[1]

2 System Under Study

2.1 Rotating Positioning Platform

Consider the rotating X-Y stage of Figure 1.

- k: Actuator's Stiffness [N/m]
- m: Payload's mass [kg]
- $\Omega = \dot{\theta}$: rotation speed [rad/s]
- F_u, F_v
- d_u, d_v

2.2 Equation of Motion

The system has two degrees of freedom and is thus fully described by the generalized coordinates u and v.

Let's express the kinetic energy T and the potential energy V of the mass m (neglecting the rotational energy):

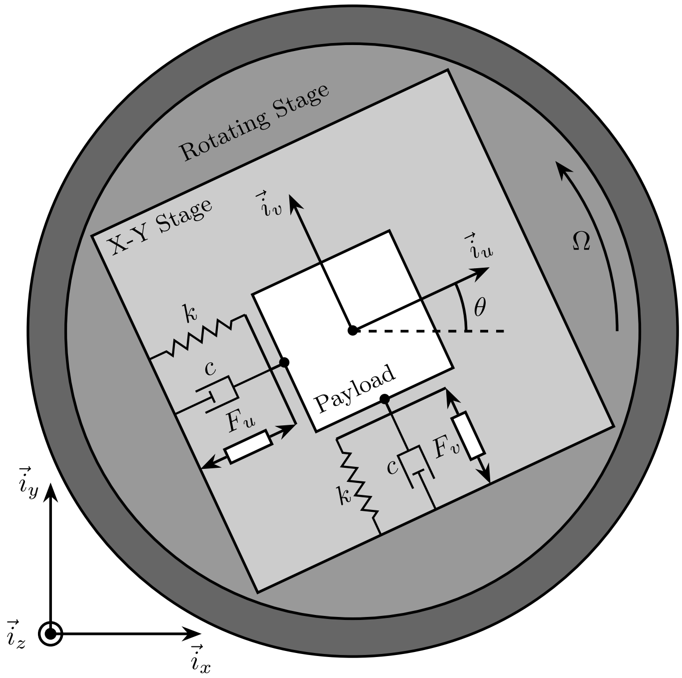


Figure 1: Figure caption

Figure 2: Figure caption

Dissipation function R Kinetic energy T Potential energy V

$$T = \frac{1}{2} m \left((\dot{u} - \Omega v)^2 + (\dot{v} + \Omega u)^2 \right) \tag{1a}$$

$$R = \frac{1}{2} c \left(\dot{u}^2 + \dot{v}^2 \right) \tag{1b}$$

$$V = \frac{1}{2} k \left(u^2 + v^2 \right) \tag{1c}$$

The Lagrangian is the kinetic energy minus the potential energy:

$$L = T - V \tag{2}$$

From the Lagrange's equations of the second kind (**??**), the equation of motion (**??**) is obtained ($q_1 = u$, $q_2 = v$).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial D}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \tag{3}$$

with Q_i is the generalized force associated with the generalized variable q_i ($Q_1 = F_u$ and $Q_2 = F_v$).

$$m\ddot{u} + c\dot{u} + (k - m\Omega)u = F_u + 2m\Omega\dot{v} \tag{4a}$$

$$m\ddot{v} + c\dot{v} + (k \underbrace{- m\Omega}_{\text{Centrif.}})v = F_v \underbrace{- 2m\Omega\dot{u}}_{\text{Coriolis}} \tag{4b}$$

- Coriolis Forces: coupling
- Centrifugal forces: negative stiffness

Without the coupling terms, each equation is the equation of a one degree of freedom mass-spring system with mass m and stiffness $k - m\dot{\theta}^2$. Thus, the term $-m\dot{\theta}^2$ acts like a negative stiffness (due to **centrifugal forces**).

2.3 Transfer Functions in the Laplace domain

$$u = \frac{ms^2 + cs + k - m\Omega^2}{\left(ms^2 + cs + k - m\Omega^2\right)^2 + \left(2m\Omega s\right)^2} F_u + \frac{2m\Omega s}{\left(ms^2 + cs + k - m\Omega^2\right)^2 + \left(2m\Omega s\right)^2} F_v \tag{5a}$$

$$v = \frac{-2m\Omega s}{\left(ms^2 + cs + k - m\Omega^2\right)^2 + \left(2m\Omega s\right)^2} F_u + \frac{ms^2 + cs + k - m\Omega^2}{\left(ms^2 + cs + k - m\Omega^2\right)^2 + \left(2m\Omega s\right)^2} F_v \tag{5b}$$

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \boldsymbol{G}_d \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{6}$$

Where $\boldsymbol{G}_d$ is a 2×2 transfer function matrix.

$$\boldsymbol{G}_d = \frac{1}{k} \frac{1}{G_{dp}} \begin{bmatrix} G_{dz} & G_{dc} \\ -G_{dc} & G_{dz} \end{bmatrix} \tag{7}$$

With:

$$G_{dp} = \left(\frac{s^2}{{\omega_0}^2} + 2\xi \frac{s}{\omega_0} + 1 - \frac{\Omega^2}{{\omega_0}^2} \right)^2 + \left(2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \right)^2 \tag{8a}$$

$$G_{dz} = \frac{s^2}{{\omega_0}^2} + 2\xi \frac{s}{\omega_0} + 1 - \frac{\Omega^2}{{\omega_0}^2} \tag{8b}$$

$$G_{dc} = 2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \tag{8c}$$

- $\omega_0 = \sqrt{\frac{k}{m}}$: Natural frequency of the mass-spring system in $\mathrm{rad/s}$
- ξ damping ratio

2.4 Constant Rotating Speed

To simplify, let's consider a constant rotating speed $\dot{\theta} = \Omega$ and thus $\ddot{\theta} = 0$.

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{1}{(ms^2 + (k - m{\omega_0}^2))^2 + (2m\omega_0 s)^2} \begin{bmatrix} ms^2 + (k - m{\omega_0}^2) & 2m\omega_0 s \\ -2m\omega_0 s & ms^2 + (k - m{\omega_0}^2) \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{\frac{1}{k}}{\left(\frac{s^2}{{\omega_0}^2} + (1 - \frac{\Omega^2}{{\omega_0}^2}) \right)^2 + \left(2 \frac{\Omega s}{{\omega_0}^2} \right)^2} \begin{bmatrix} \frac{s^2}{{\omega_0}^2} + 1 - \frac{\Omega^2}{{\omega_0}^2} & 2 \frac{\Omega s}{{\omega_0}^2} \\ -2 \frac{\Omega s}{{\omega_0}^2} & \frac{s^2}{{\omega_0}^2} + 1 - \frac{\Omega^2}{{\omega_0}^2} \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{10}$$

When the rotation speed is null, the coupling terms are equal to zero and the diagonal terms corresponds to one degree of freedom mass spring system.

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{\frac{1}{k}}{\frac{s^2}{{\omega_0}^2} + 1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{11}$$

When the rotation speed in not null, the resonance frequency is duplicated into two pairs of complex conjugate poles. As the rotation speed increases, one of the two resonant frequency goes to lower frequencies as the other one goes to higher frequencies (Figure 3).

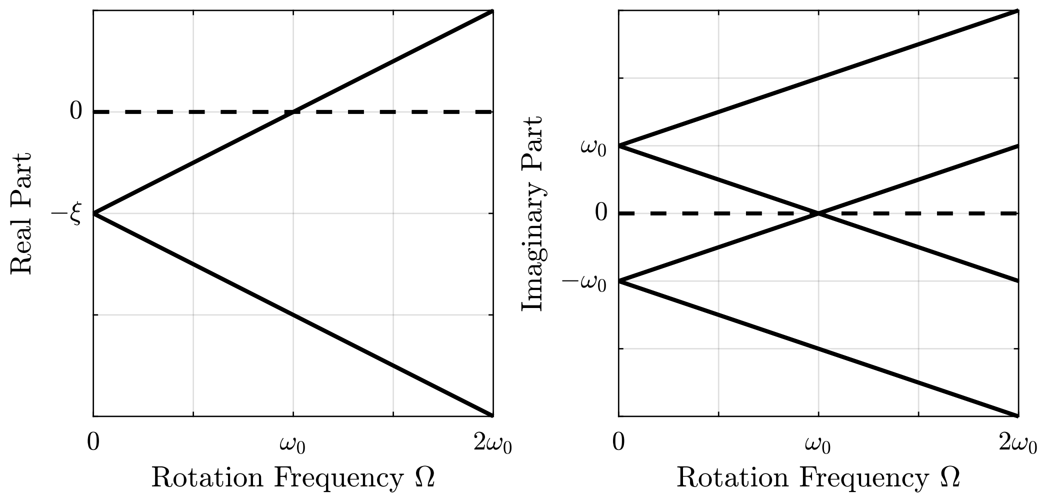


Figure 3: Campbell Diagram

The magnitude of the coupling terms are increasing with the rotation speed.

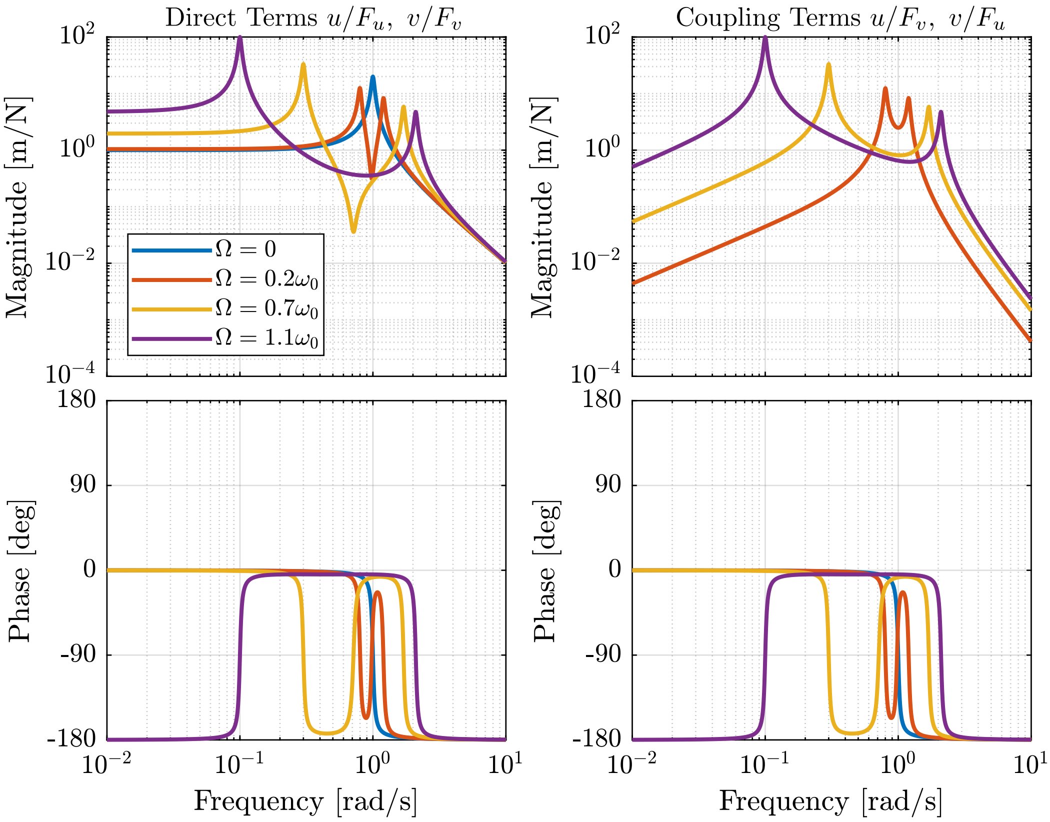


Figure 4: Caption

3 Integral Force Feedback

3.1 Control Schematic

Force Sensors are added in series with the actuators as shown in Figure 5.

3.2 Equations

The sensed forces are equal to:

$$\begin{bmatrix} f_u \\ f_v \end{bmatrix} = \begin{bmatrix} F_u \\ F_v \end{bmatrix} - (cs + k) \begin{bmatrix} d_u \\ d_v \end{bmatrix} \tag{12}$$

Which then gives:

$$\begin{bmatrix} f_u \\ f_v \end{bmatrix} = \boldsymbol{G}_f \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} f_u \\ f_v \end{bmatrix} = \frac{1}{G_{fp}} \begin{bmatrix} G_{fz} & -G_{fc} \\ G_{fc} & G_{fz} \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{14}$$

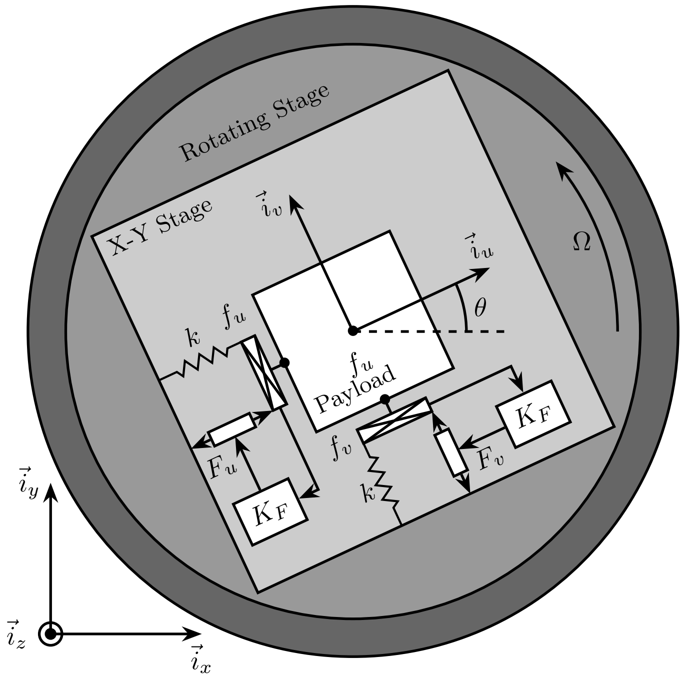


Figure 5: System with Force Sensors in Series with the Actuators. Decentralized Integral Force Feedback is used

$$G_{fp} = \left(\frac{s^2}{{\omega_0}^2} + 2\xi\frac{s}{\omega_0} + 1 - \frac{\Omega^2}{{\omega_0}^2}\right)^2 + \left(2\frac{\Omega}{\omega_0}\frac{s}{\omega_0}\right)^2 \tag{15}$$

$$G_{fz} = \left(\frac{s^2}{{\omega_0}^2} - \frac{\Omega^2}{{\omega_0}^2}\right)\left(\frac{s^2}{{\omega_0}^2} + 2\xi\frac{s}{\omega_0} + 1 - \frac{\Omega^2}{{\omega_0}^2}\right) + \left(2\frac{\Omega}{\omega_0}\frac{s}{\omega_0}\right)^2 \tag{16}$$

$$G_{fc} = \left(2\xi\frac{s}{\omega_0} + 1\right)\left(2\frac{\Omega}{\omega_0}\frac{s}{\omega_0}\right) \tag{17}$$

3.3 Plant Dynamics

3.4 Physical Interpretation

At low frequency, the gain is very large and thus no force is transmitted between the payload and the rotating stage. This means that at low frequency, the system is decoupled (the force sensor removed) and thus the system is unstable.

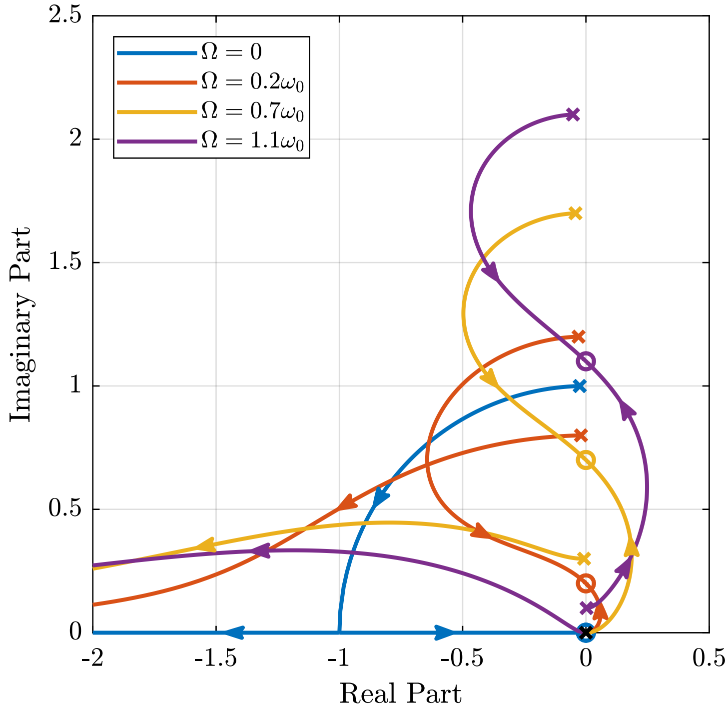


Figure 6: Root Locus

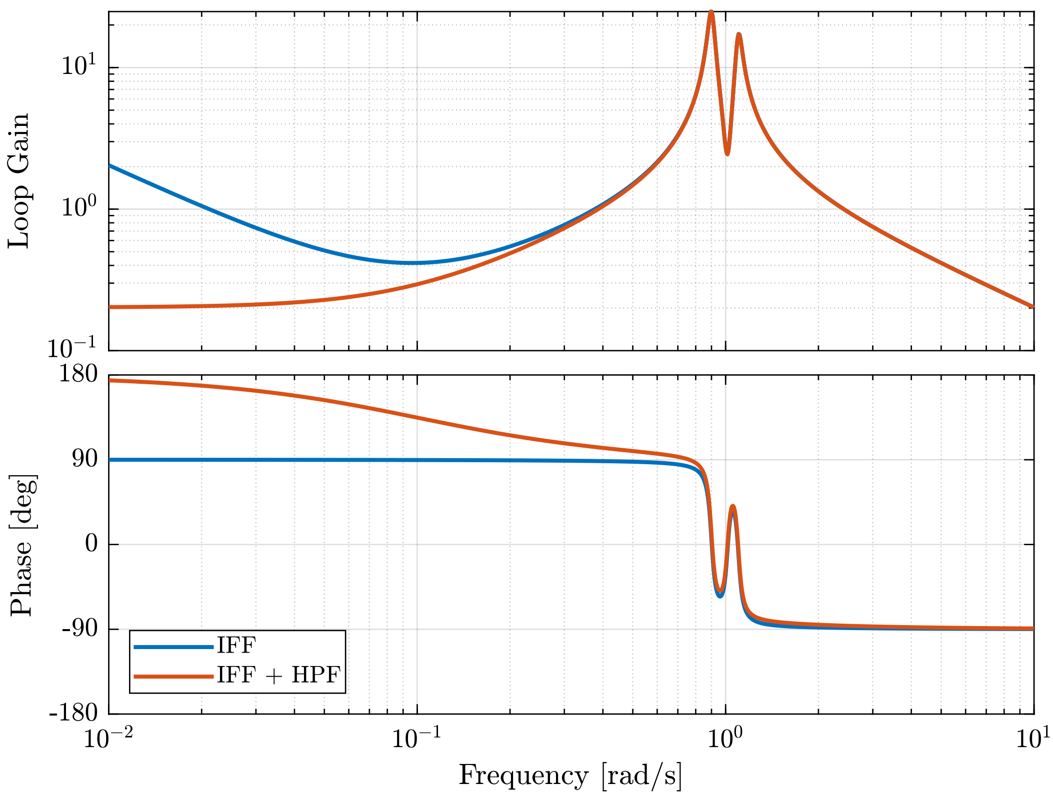
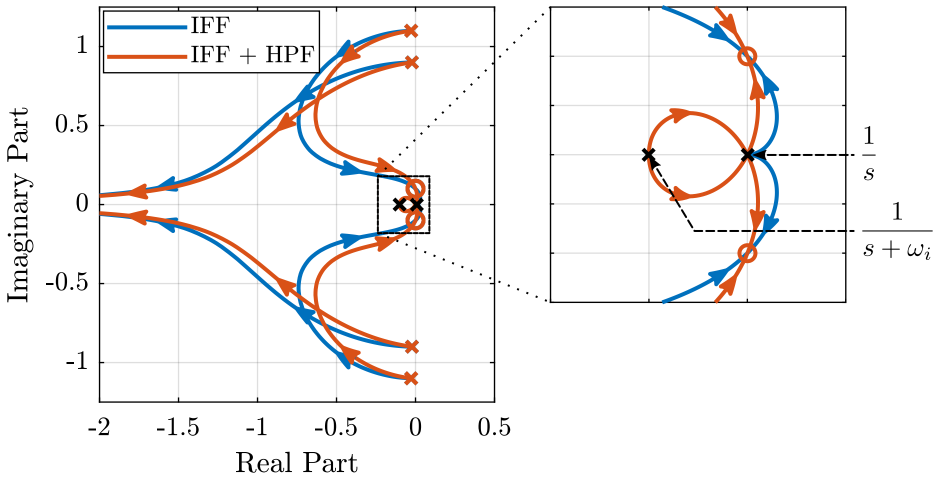


Figure 7: Figure caption



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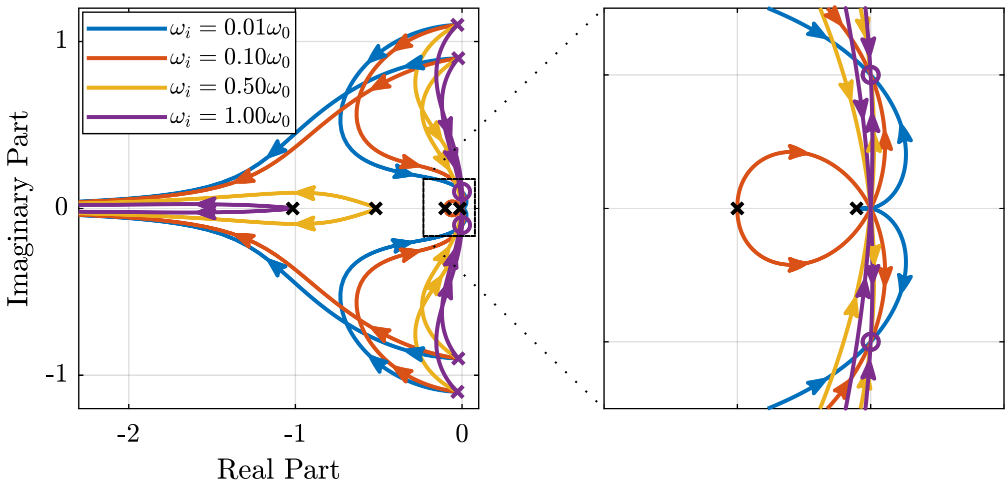

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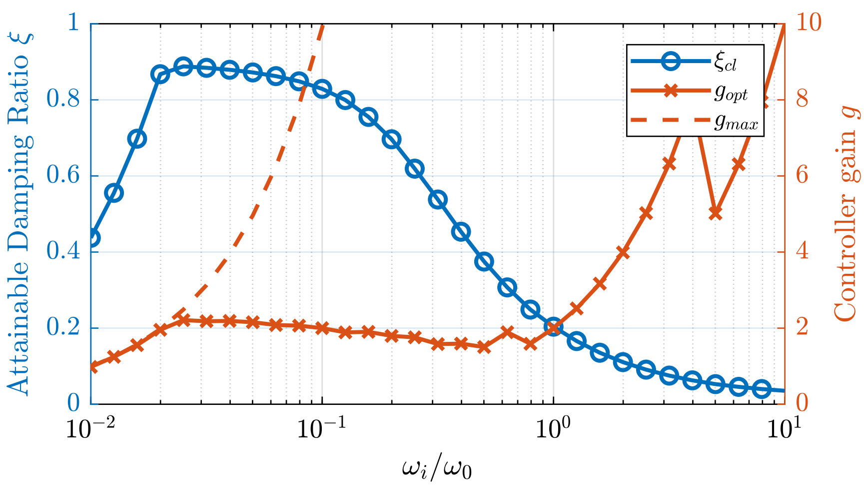

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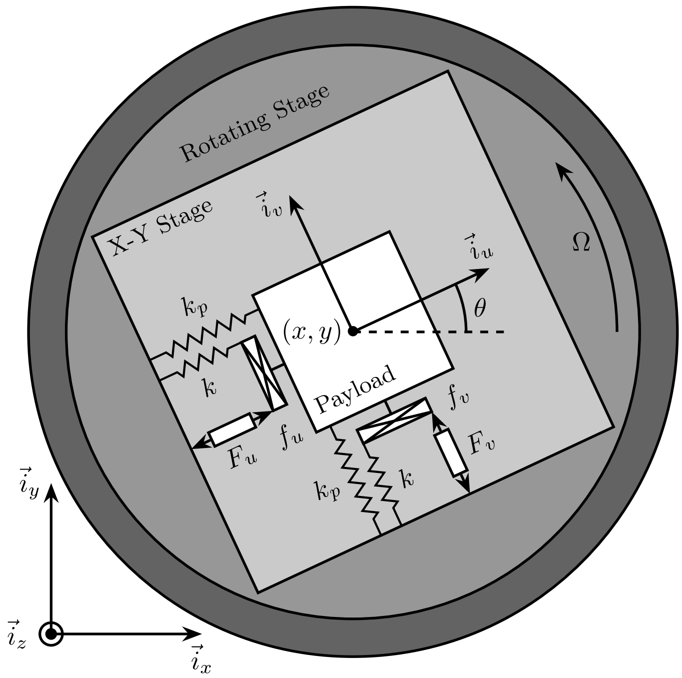

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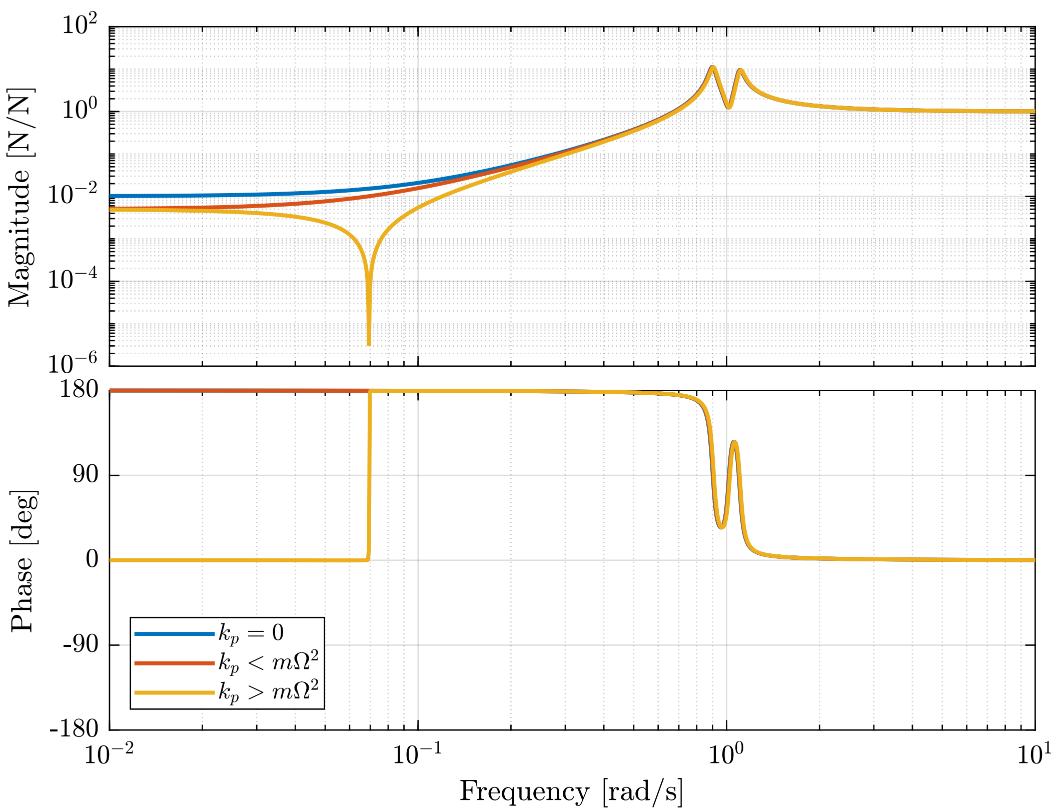

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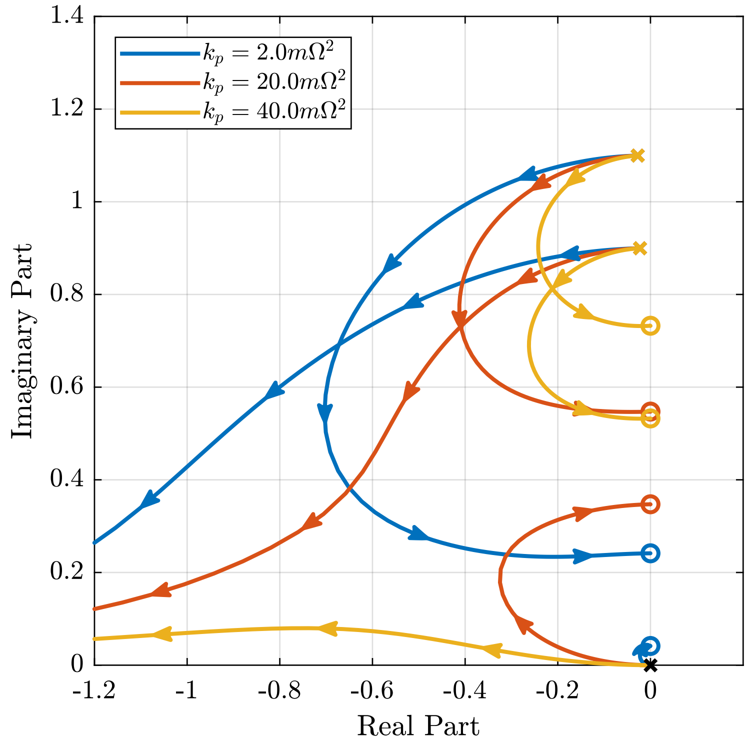

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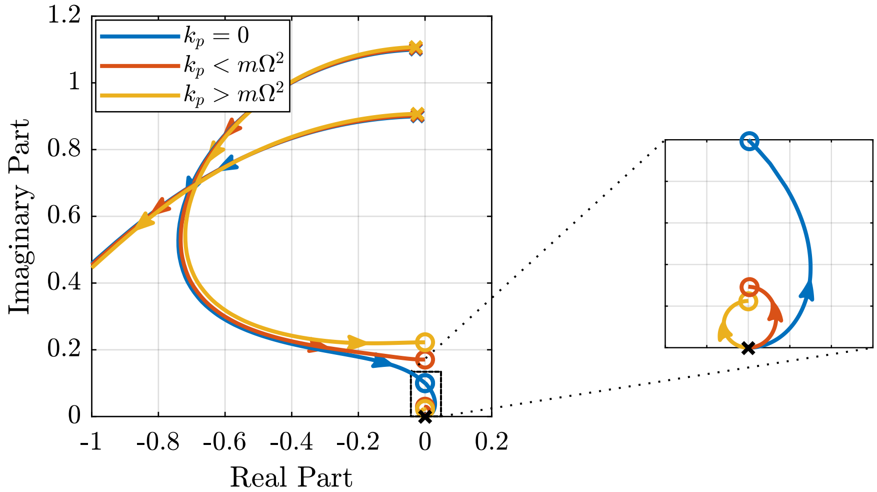

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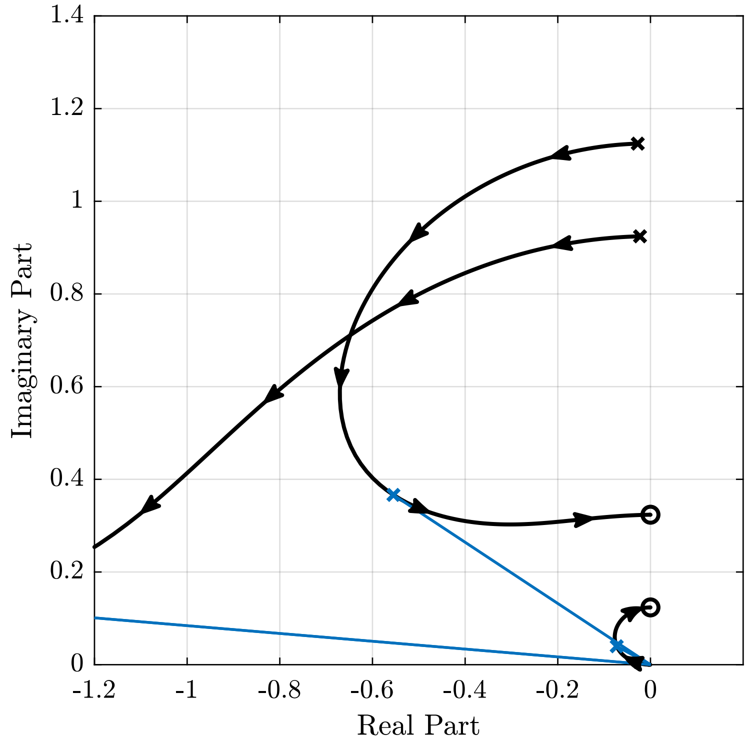

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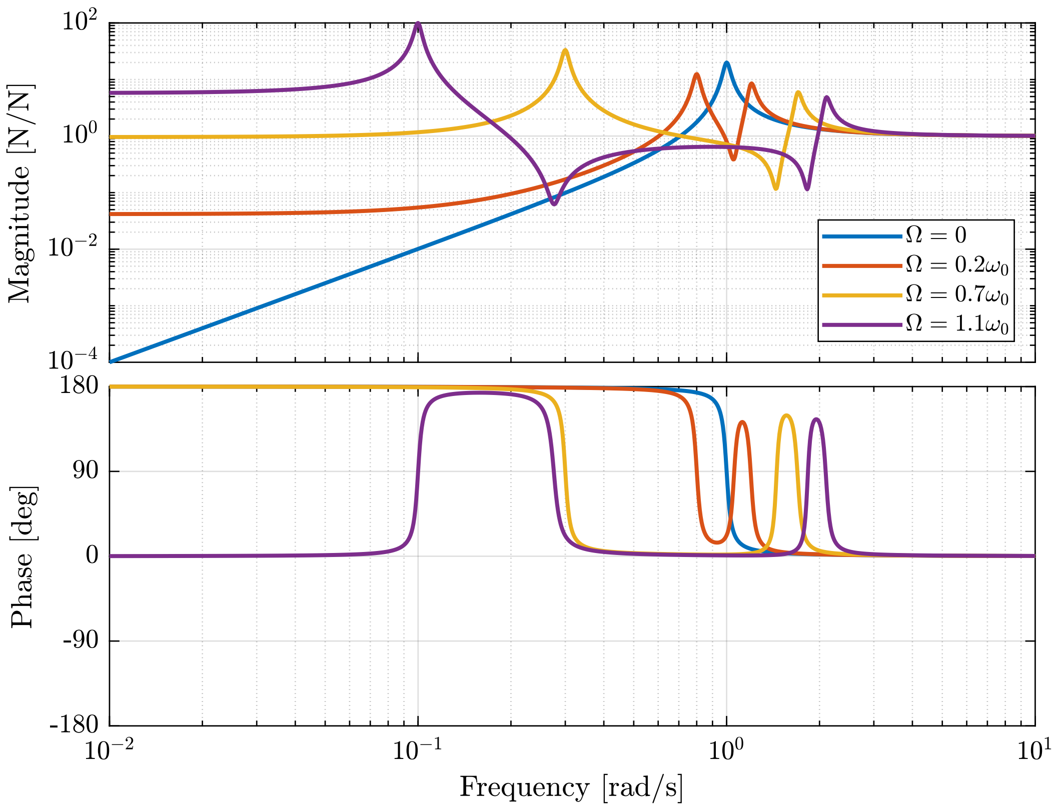


Figure 16: Figure caption

4 Integral Force Feedback with High Pass Filters

4.1 Modification of the Control Low

4.2 Close Loop Analysis

4.3 Optimal Cut-Off Frequency

5 Integral Force Feedback with Parallel Springs

6 Direct Velocity Feedback

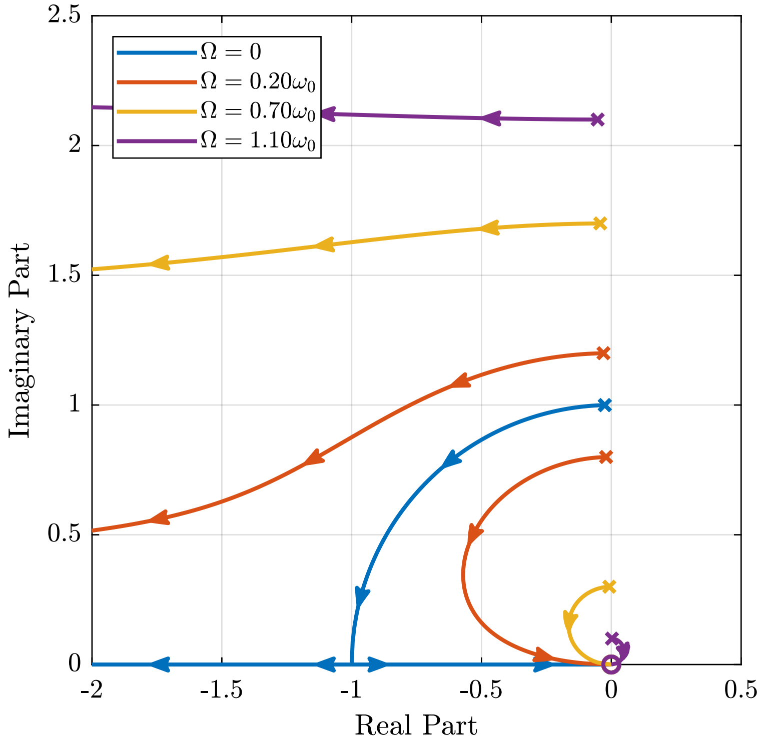


Figure 17: Figure caption

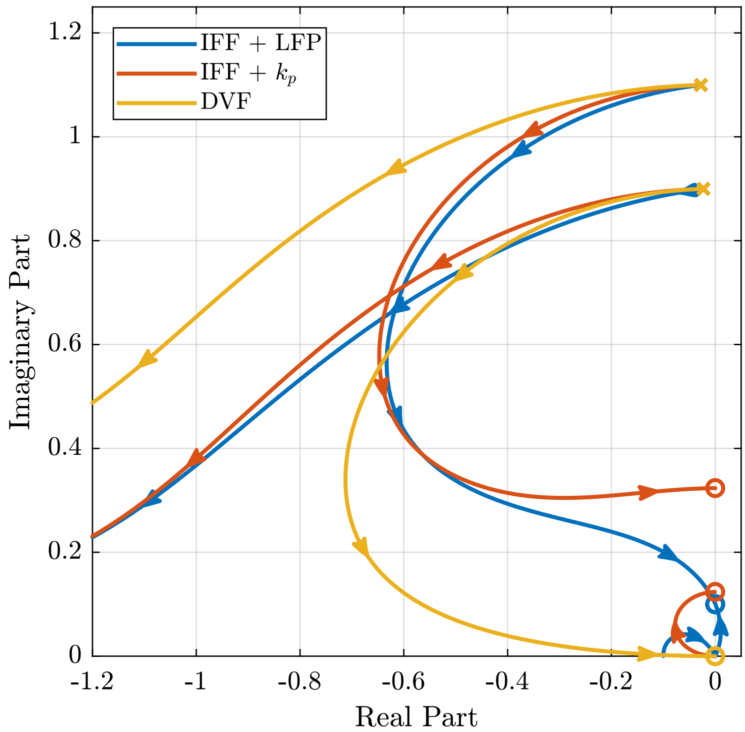
7 Comparison of the Proposed Active Damping Techniques

8 Conclusion

Acknowledgment

References

[1] T. Dehaeze, M. M. Mattenet, and C. Collette, "Sample stabilization for tomography experiments in presence of large plant uncertainty," in *MEDSI'18*, ser. Mechanical Engineering Design of Synchrotron Radiation Equipment and Instrumentation, no. 10. Geneva, Switzerland: JACoW Publishing, Dec 2018, pp. 153–157. [Online]. Available: https://doi.org/10.18429/JACoW-MEDSI2018-WEOAMA02



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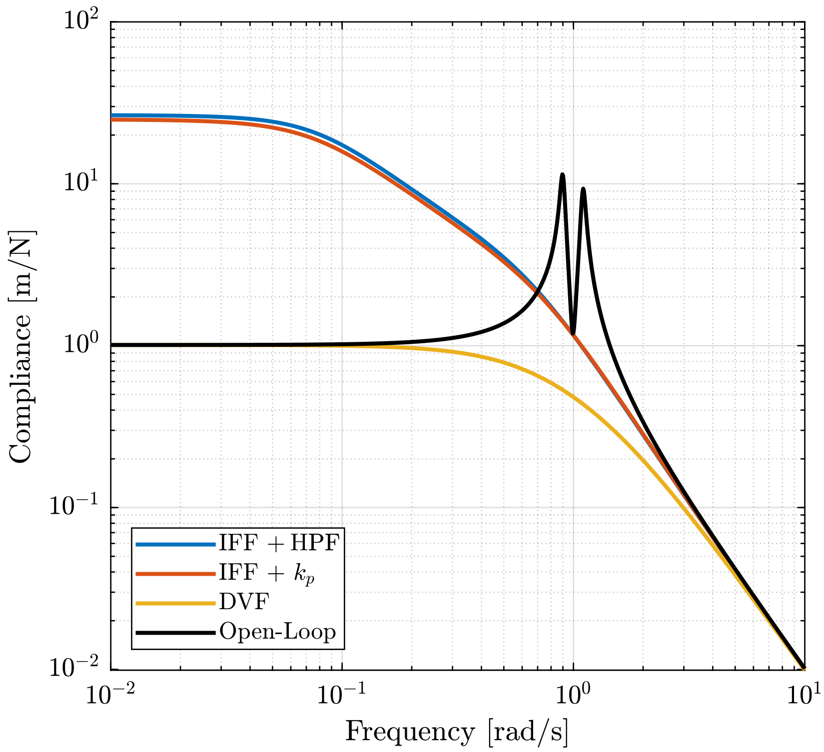

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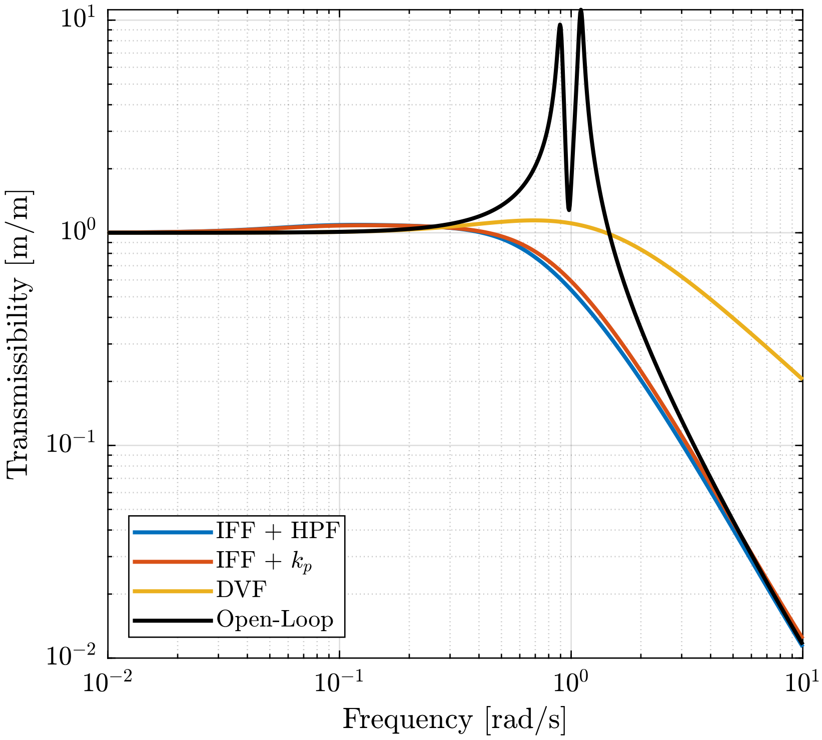

Figure 20: Figure caption