Active Damping of Rotating Positioning Platforms

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Abstract
Abstract text to be done

1 Introduction

[1]

2 System Under Study

2.1 Rotating Positioning Platform

Consider the rotating X-Y stage of Figure 1.

- k: Actuator's Stiffness [N/m]
- m: Payload's mass [kg]
- $\Omega = \dot{\theta}$: rotation speed [rad/s]
- F_u, F_v
- d_u, d_v

2.2 Equation of Motion

The system has two degrees of freedom and is thus fully described by the generalized coordinates u and v.

Let's express the kinetic energy T and the potential energy V of the mass m (neglecting the rotational energy):

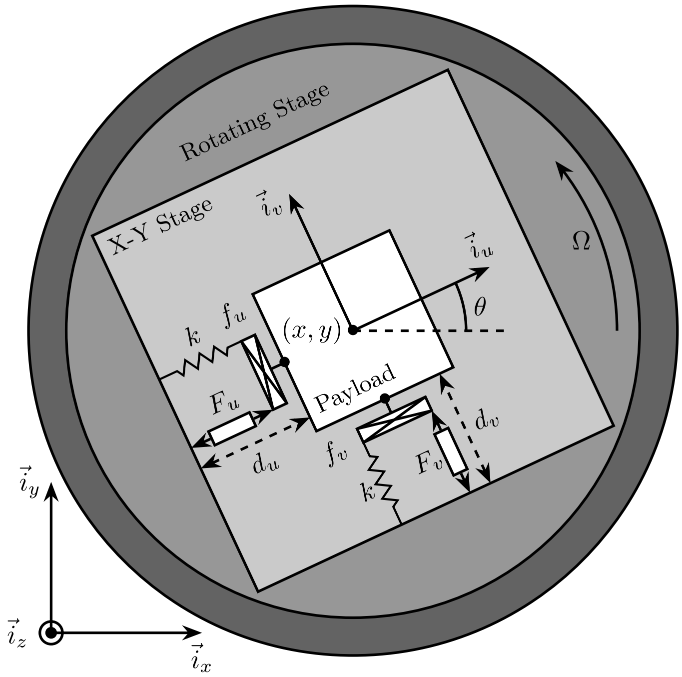


Figure 1: Figure caption

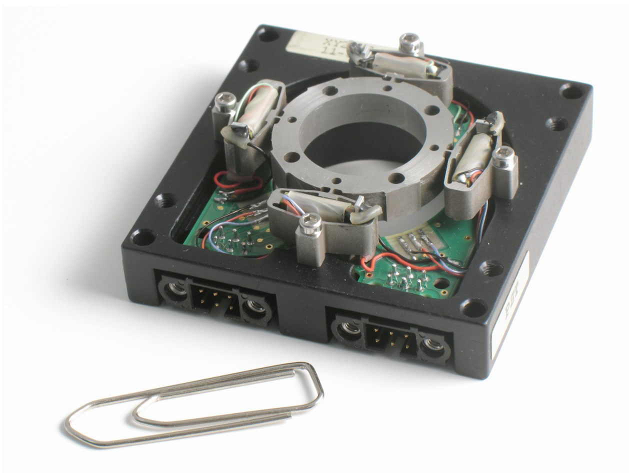


Figure 2: Figure caption

Dissipation function R Kinetic energy T Potential energy V

$$T = \frac{1}{2} m \left((\dot{u} - \Omega v)^2 + (\dot{v} + \Omega u)^2 \right) \tag{1a}$$
$$R = \frac{1}{2} c \left(\dot{u}^2 + \dot{v}^2 \right) \tag{1b}$$
$$V = \frac{1}{2} k \left(u^2 + v^2 \right) \tag{1c}$$

The Lagrangian is the kinetic energy minus the potential energy:

$$L = T - V \tag{2}$$

From the Lagrange's equations of the second kind (10), the equation of motion (11) is obtained ($q_1 = u$, $q_2 = v$).

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) + \frac{\partial D}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \tag{3}$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} - \frac{\partial V}{\partial q_i} = Q_i \tag{4}$$

with Q_i is the generalized force associated with the generalized variable q_i (F_u and F_v).

$$m\ddot{u} + c\dot{u} + (k - m\Omega)u = F_u + 2m\Omega\dot{v} \tag{5a}$$
$$m\ddot{v} + c\dot{v} + (k \underbrace{- m\Omega}_{\text{Centrif.}})v = F_v \underbrace{- 2m\Omega\dot{u}}_{\text{Coriolis}} \tag{5b}$$

$$u = \frac{ms^2 + cs + k - m\Omega^2}{\left(ms^2 + cs + k - m\Omega^2\right)^2 + (2m\Omega s)^2} F_u + \frac{2m\Omega s}{\left(ms^2 + cs + k - m\Omega^2\right)^2 + (2m\Omega s)^2} F_v \tag{6a}$$
$$v = \frac{-2m\Omega s}{\left(ms^2 + cs + k - m\Omega^2\right)^2 + (2m\Omega s)^2} F_u + \frac{ms^2 + cs + k - m\Omega^2}{\left(ms^2 + cs + k - m\Omega^2\right)^2 + (2m\Omega s)^2} F_v \tag{6b}$$

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \boldsymbol{G}_d \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{7}$$

Where $\boldsymbol{G}_d$ is a 2×2 transfer function matrix.

$$\boldsymbol{G}_d = \frac{1}{k} \frac{1}{G_{dp}} \begin{bmatrix} G_{dz} & G_{dc} \\ -G_{dc} & G_{dz} \end{bmatrix} \tag{8}$$

With:

$$G_{dp} = \left(\frac{s^2}{{\omega_0}^2} + 2\xi \frac{s}{\omega_0} + 1 - \frac{\Omega^2}{{\omega_0}^2} \right)^2 + \left(2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \right)^2 \tag{9a}$$
$$G_{dz} = \frac{s^2}{{\omega_0}^2} + 2\xi \frac{s}{\omega_0} + 1 - \frac{\Omega^2}{{\omega_0}^2} \tag{9b}$$
$$G_{dc} = 2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \tag{9c}$$

- $\omega_0 = \sqrt{\frac{k}{m}}$: Natural frequency of the mass-spring system in rad/s
- ξ damping ratio

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = \frac{\partial L}{\partial q_j} \tag{10}$$

$$m\ddot{x} + kx = F_u \cos\theta - F_v \sin\theta \tag{11a}$$
$$m\ddot{y} + ky = F_u \sin\theta + F_v \cos\theta \tag{11b}$$

Performing the change coordinates from (x, y) to (d_x, d_y, θ):

$$x = d_u \cos\theta - d_v \sin\theta \tag{12a}$$
$$y = d_u \sin\theta + d_v \cos\theta \tag{12b}$$

Gives

$$m\ddot{d}_u + (k - m\dot{\theta}^2)d_u = F_u + 2m\dot{d}_v\dot{\theta} + md_v\ddot{\theta} \tag{13a}$$
$$m\ddot{d}_v + (k \underbrace{- m\dot{\theta}^2}_{\text{Centrif.}})d_v = F_v \underbrace{- 2m\dot{d}_u\dot{\theta}}_{\text{Coriolis}} \underbrace{- md_u\ddot{\theta}}_{\text{Euler}} \tag{13b}$$

We obtain two differential equations that are coupled through:

- **Euler forces**: $md_v\ddot{\theta}$
- **Coriolis forces**: $2m\dot{d}_v\dot{\theta}$

Without the coupling terms, each equation is the equation of a one degree of freedom mass-spring system with mass m and stiffness $k - m\dot{\theta}^2$. Thus, the term $-m\dot{\theta}^2$ acts like a negative stiffness (due to **centrifugal forces**).

2.3 Constant Rotating Speed

To simplify, let's consider a constant rotating speed $\dot{\theta} = \Omega$ and thus $\ddot{\theta} = 0$.

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{1}{(ms^2 + (k - m{\omega_0}^2))^2 + (2m\omega_0 s)^2} \begin{bmatrix} ms^2 + (k - m{\omega_0}^2) & 2m\omega_0 s \\ -2m\omega_0 s & ms^2 + (k - m{\omega_0}^2) \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{\frac{1}{k}}{\left(\frac{s^2}{{\omega_0}^2} + (1 - \frac{\Omega^2}{{\omega_0}^2})\right)^2 + \left(2\frac{\Omega s}{{\omega_0}^2}\right)^2} \begin{bmatrix} \frac{s^2}{{\omega_0}^2} + 1 - \frac{\Omega^2}{{\omega_0}^2} & 2\frac{\Omega s}{{\omega_0}^2} \\ -2\frac{\Omega s}{{\omega_0}^2} & \frac{s^2}{{\omega_0}^2} + 1 - \frac{\Omega^2}{{\omega_0}^2} \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{15}$$

When the rotation speed is null, the coupling terms are equal to zero and the diagonal terms corresponds to one degree of freedom mass spring system.

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{\frac{1}{k}}{\frac{s^2}{{\omega_0}^2} + 1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{16}$$

When the rotation speed in not null, the resonance frequency is duplicated into two pairs of complex conjugate poles. As the rotation speed increases, one of the two resonant frequency goes to lower frequencies as the other one goes to higher frequencies (Figure 3).

The magnitude of the coupling terms are increasing with the rotation speed.

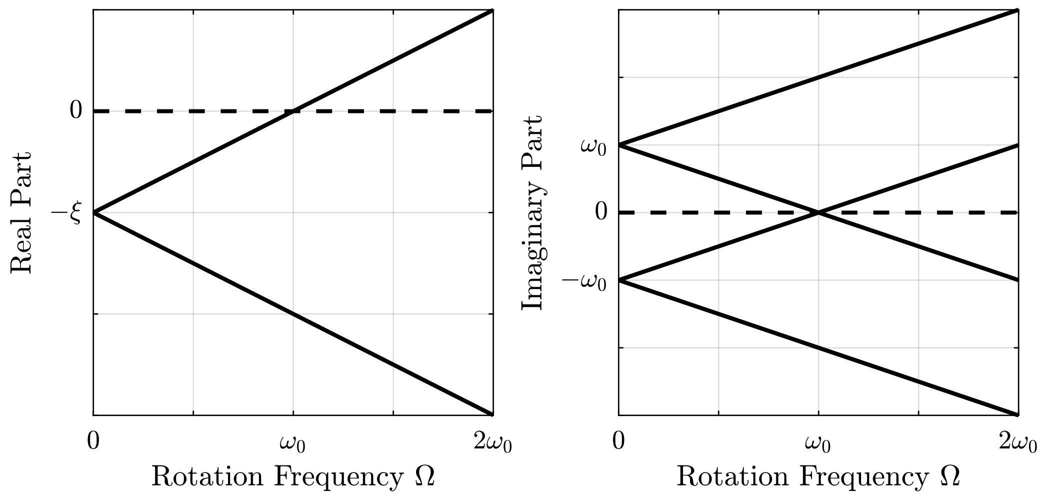


Figure 3: Campbell Diagram

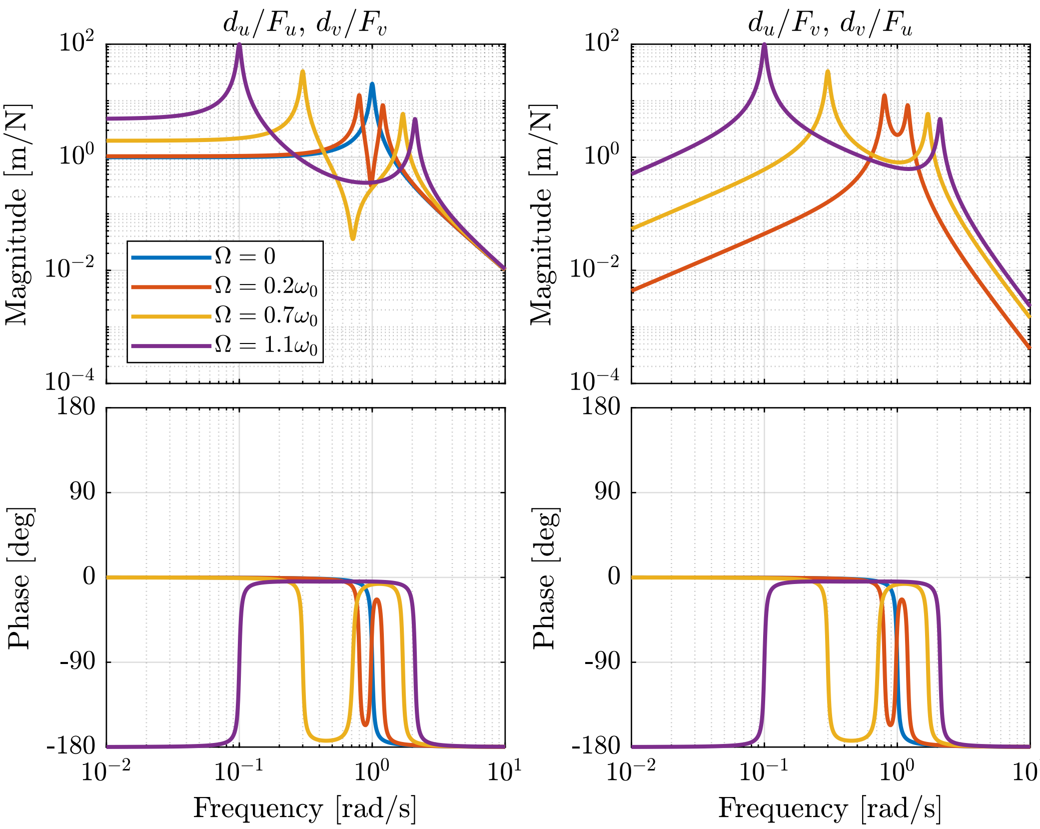


Figure 4: Caption

3 Integral Force Feedback

3.1 Control Schematic

3.2 Equations

3.3 Plant Dynamics

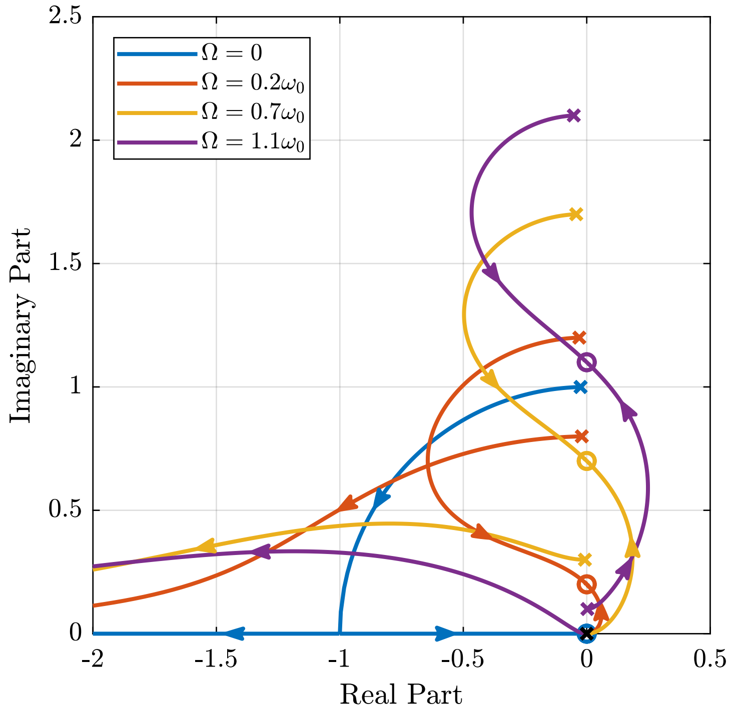


Figure 5: Figure caption

3.4 Physical Interpretation

4 Integral Force Feedback with Low Pass Filters

5 Integral Force Feedback with Parallel Springs

6 Direct Velocity Feedback

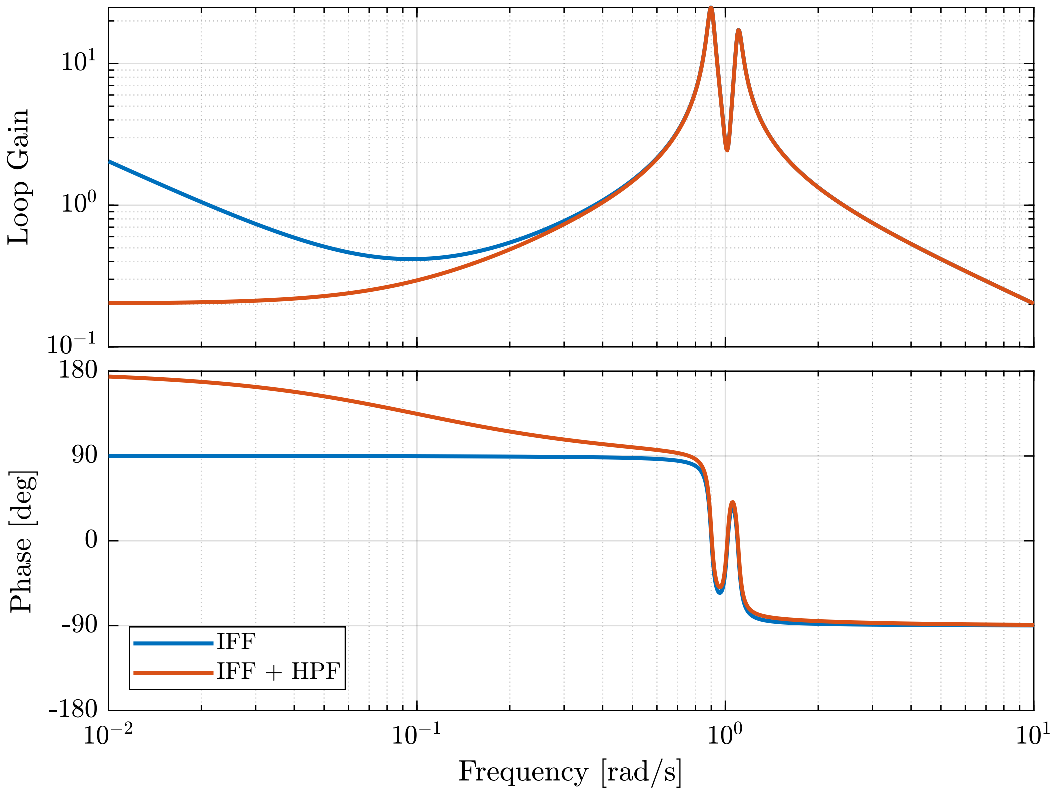
7 Comparison of the Proposed Active Damping Techniques

8 Conclusion

Acknowledgment

References

[1] T. Dehaeze, M. M. Mattenet, and C. Collette, "Sample stabilization for tomography experiments in presence of large plant uncertainty," in *MEDSI'18*, ser. Mechanical Engineering Design of Synchrotron



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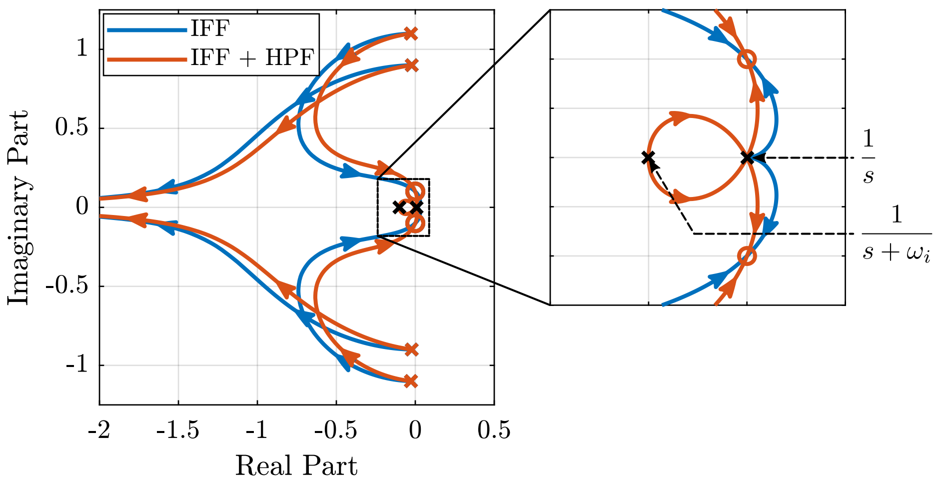

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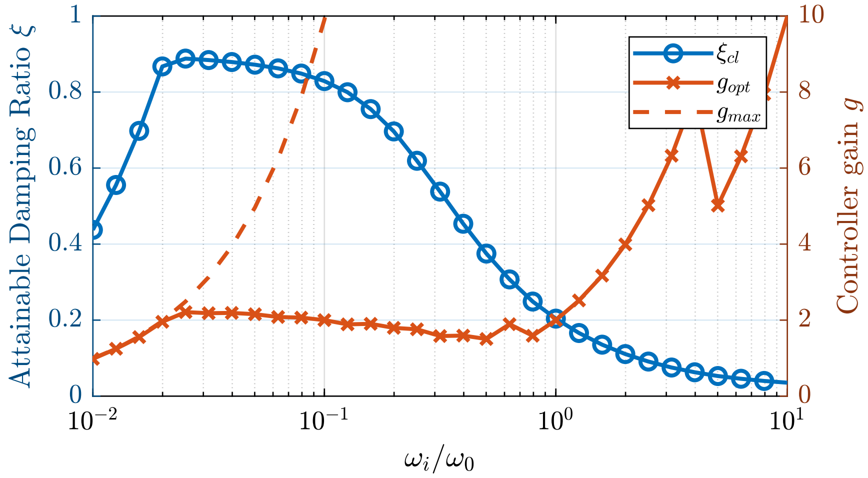

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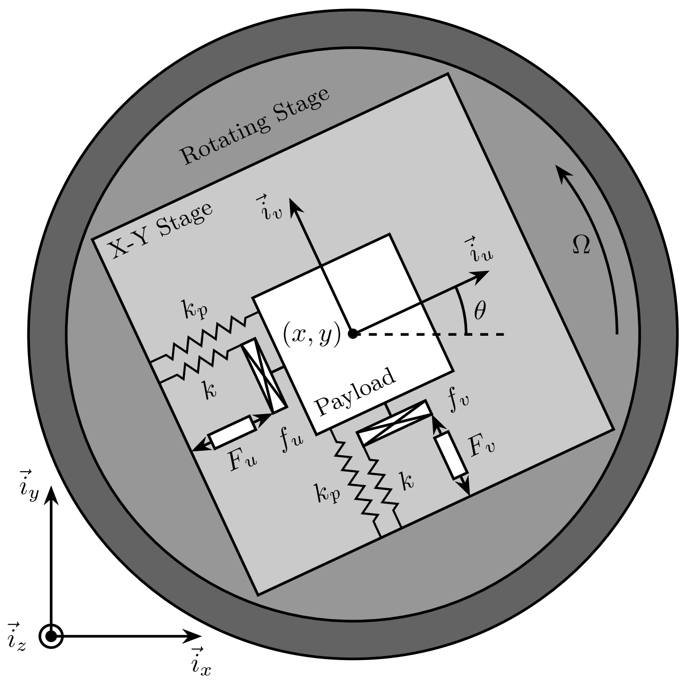

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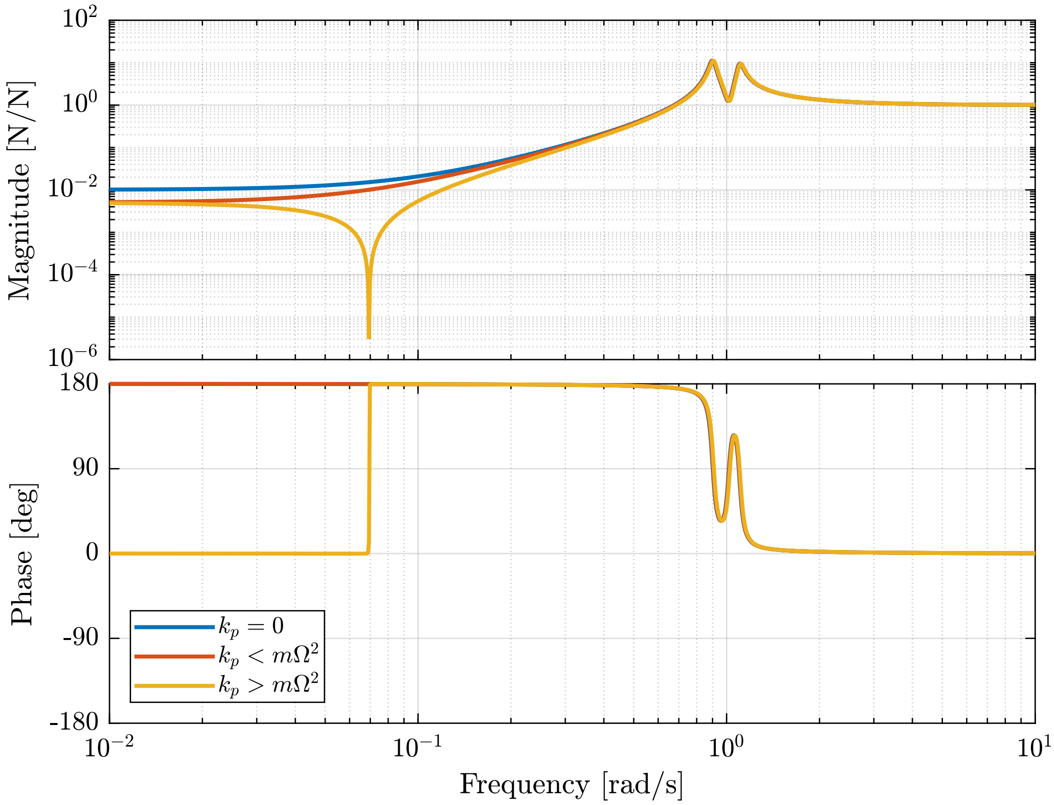

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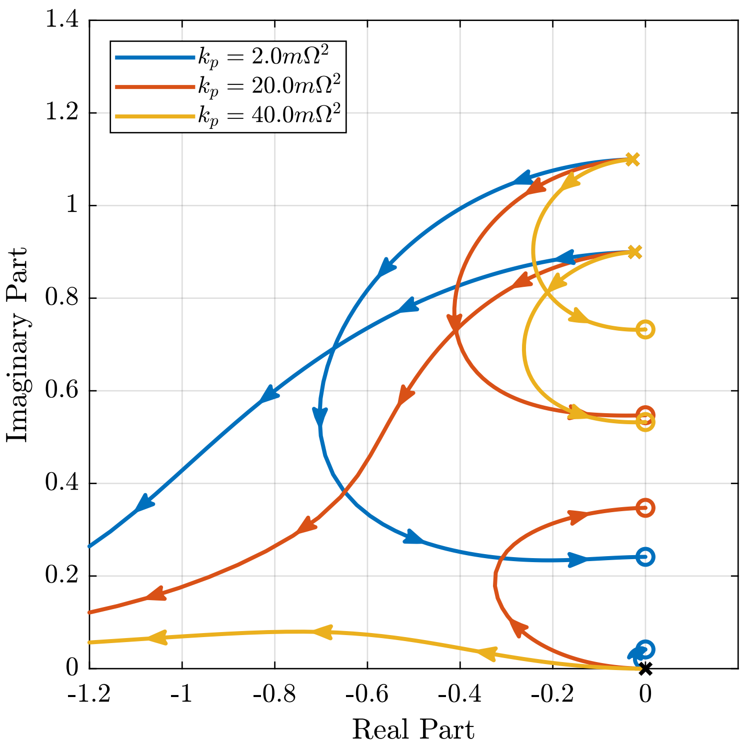

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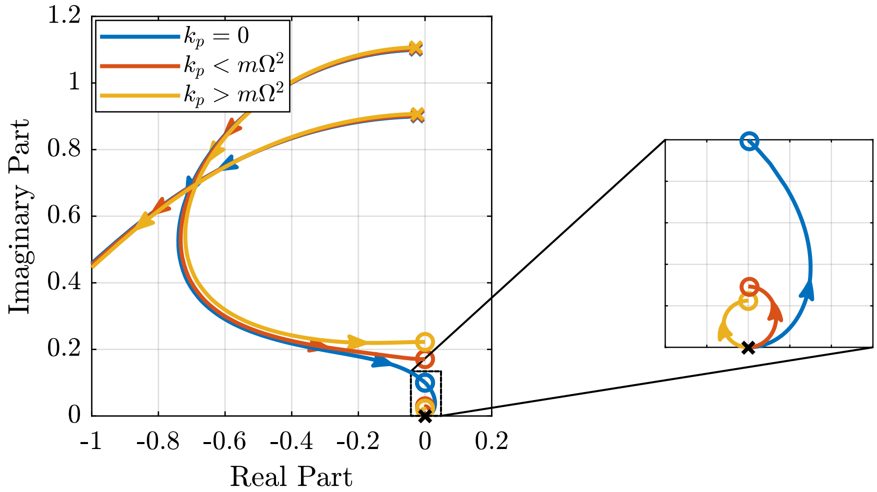

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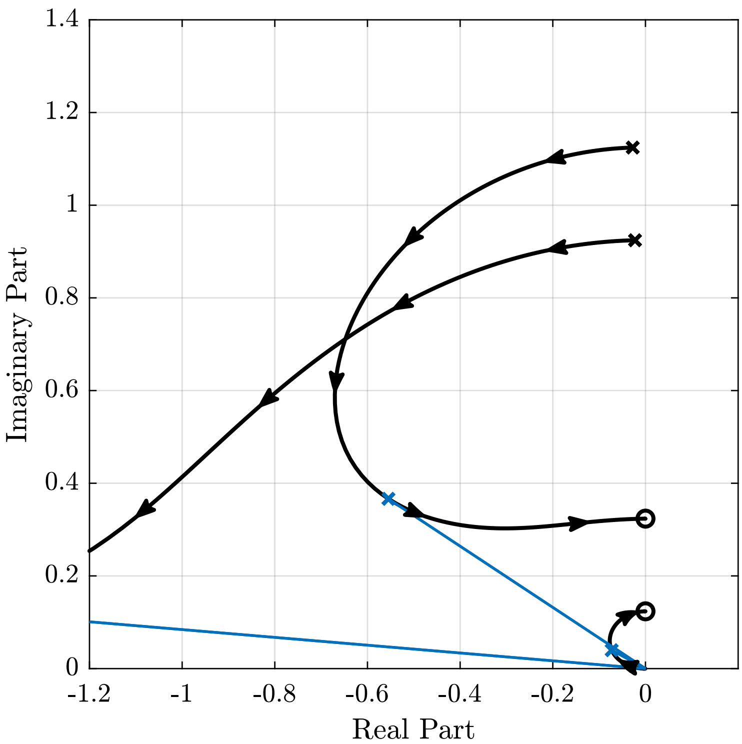

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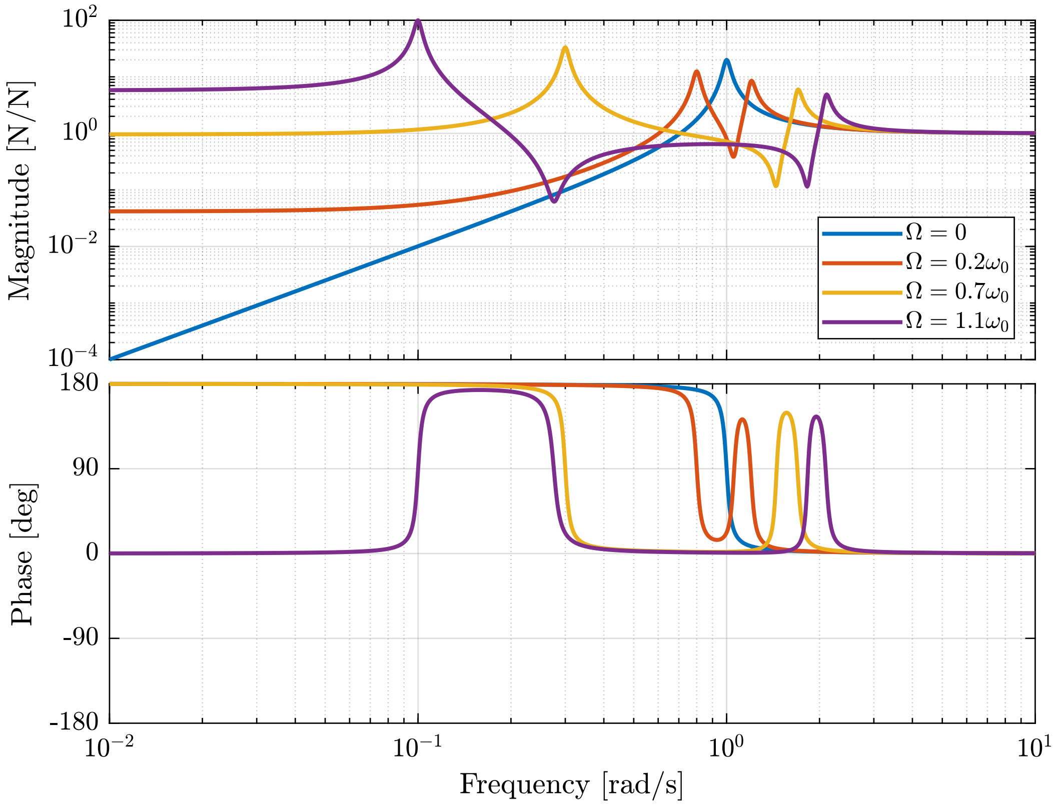

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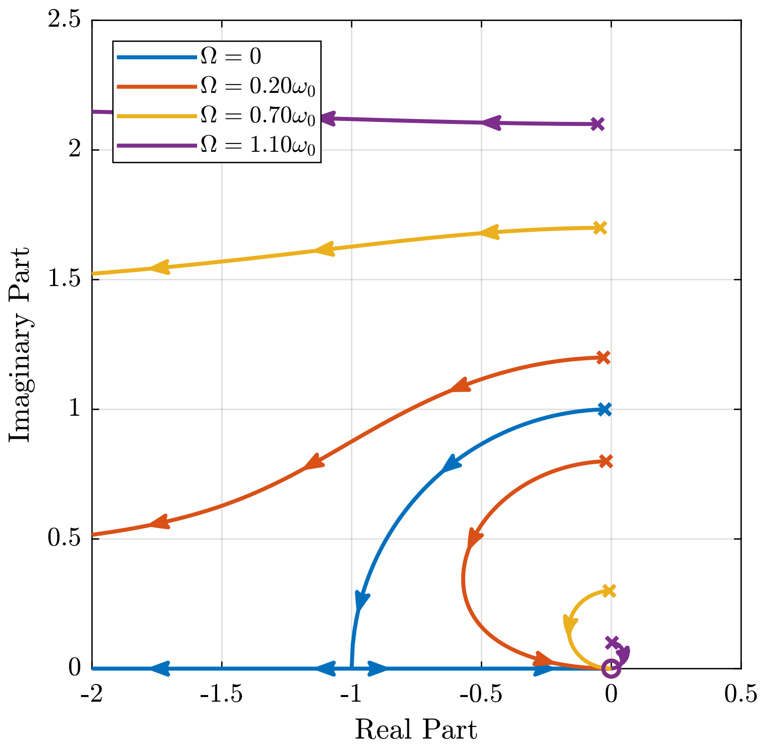

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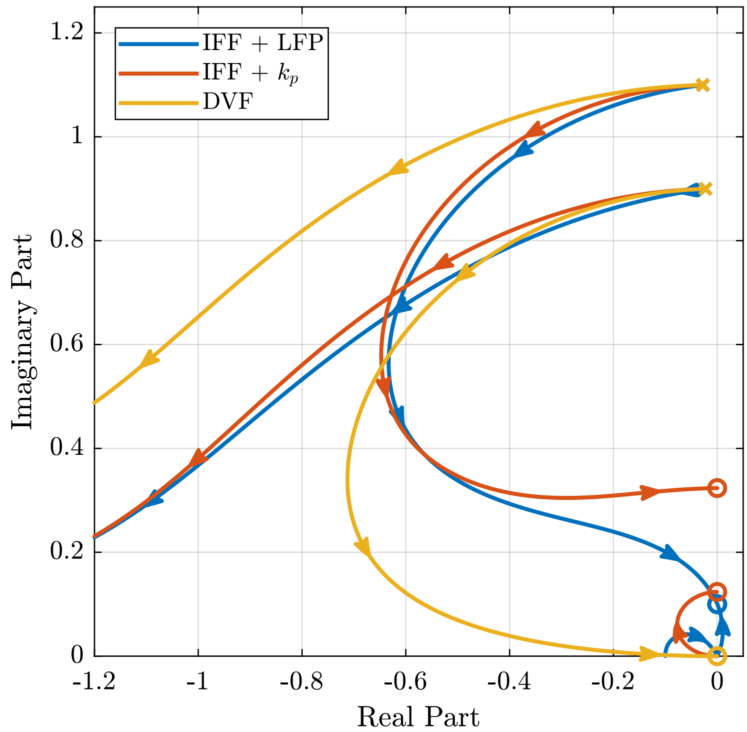

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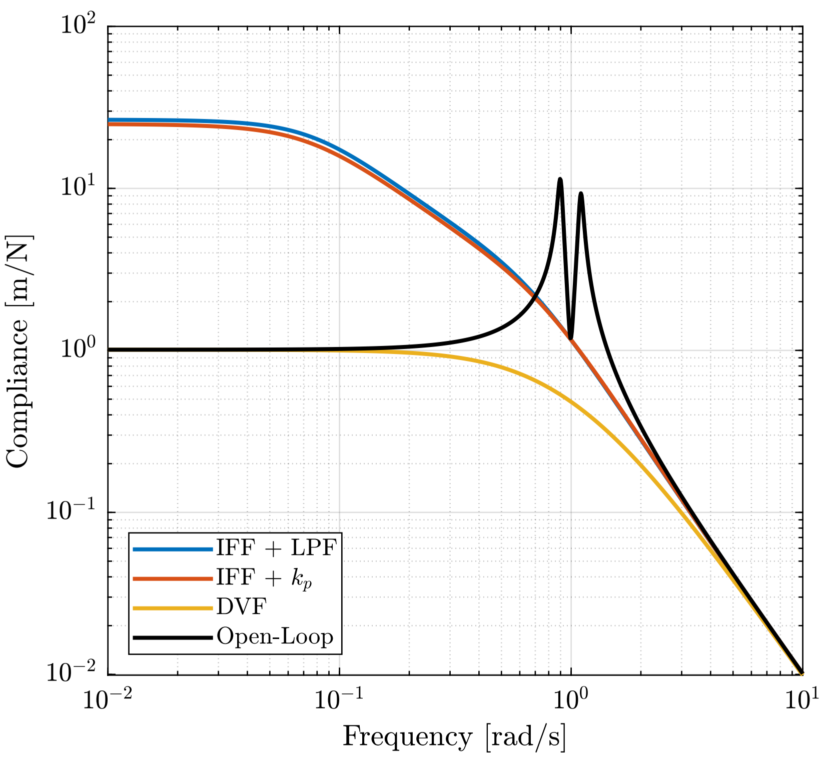

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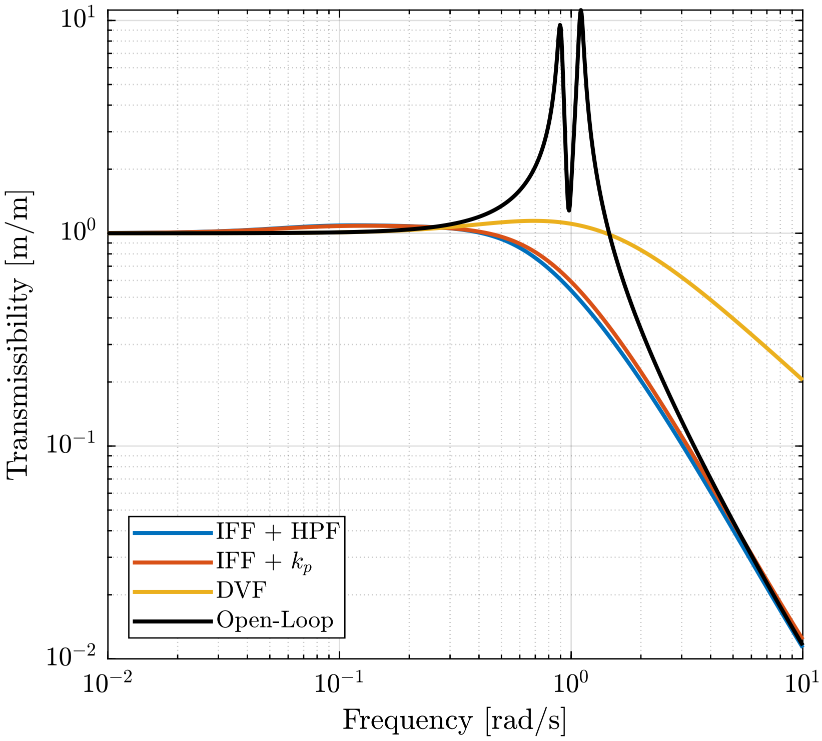

Figure 18: Figure caption

Radiation Equipment and Instrumentation, no. 10. Geneva, Switzerland: JACoW Publishing, Dec 2018, pp. 153–157. [Online]. Available: https://doi.org/10.18429/JACoW-MEDSI2018-WEOAMA02