

Active Damping of Rotating Positioning Platforms

T. Dehaeze^{1,3}, C. Collette^{1,2}

¹ Precision Mechatronics Laboratory
University of Liege, Belgium

² BEAMS Department
Free University of Brussels, Belgium

³ European Synchrotron Radiation Facility
Grenoble, France e-mail: thomas.dehaeze@esrf.fr

Abstract

Abstract text to be done

1 Introduction

Controller Poles are shown by black crosses (✕).

2 Dynamics of Rotating Positioning Platforms

2.1 Studied Rotating Positioning Platform

Consider the rotating X-Y stage of Figure 1.

- k : Actuator's Stiffness [N/m]
- m : Payload's mass [kg]
- $\Omega = \dot{\theta}$: rotation speed [rad/s]
- F_u, F_v
- d_u, d_v

2.2 Equation of Motion

The system has two degrees of freedom and is thus fully described by the generalized coordinates u and v .

Let's express the kinetic energy T and the potential energy V of the mass m (neglecting the rotational energy):

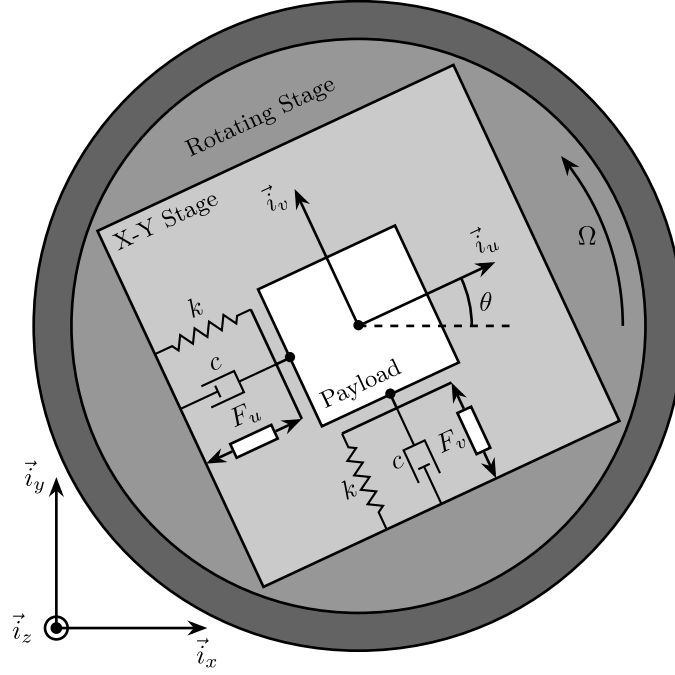


Figure 1: Figure caption

Dissipation function R Kinetic energy T Potential energy V

$$T = \frac{1}{2}m \left((\dot{u} - \Omega v)^2 + (\dot{v} + \Omega u)^2 \right) \quad (1a)$$

$$R = \frac{1}{2}c (\dot{u}^2 + \dot{v}^2) \quad (1b)$$

$$V = \frac{1}{2}k (u^2 + v^2) \quad (1c)$$

The Lagrangian is the kinetic energy minus the potential energy:

$$L = T - V \quad (2)$$

From the Lagrange's equations of the second kind, the equation of motion is obtained ($q_1 = u$, $q_2 = v$).

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) + \frac{\partial D}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \quad (3)$$

with Q_i is the generalized force associated with the generalized variable q_i ($Q_1 = F_u$ and $Q_2 = F_v$).

$$m\ddot{u} + c\dot{u} + (k - m\Omega^2)u = F_u + 2m\Omega\dot{v} \quad (4a)$$

$$m\ddot{v} + c\dot{v} + \underbrace{(k - m\Omega^2)}_{\text{Centrif.}}v = F_v - \underbrace{2m\Omega\dot{u}}_{\text{Coriolis}} \quad (4b)$$

- Coriolis Forces: coupling
- Centrifugal forces: negative stiffness

Without the coupling terms, each equation is the equation of a one degree of freedom mass-spring system with mass m and stiffness $k - m\dot{\theta}^2$. Thus, the term $-m\dot{\theta}^2$ acts like a negative stiffness (due to **centrifugal forces**).

2.3 Transfer Functions in the Laplace domain

$$u = \frac{ms^2 + cs + k - m\Omega^2}{(ms^2 + cs + k - m\Omega^2)^2 + (2m\Omega s)^2} F_u + \frac{2m\Omega s}{(ms^2 + cs + k - m\Omega^2)^2 + (2m\Omega s)^2} F_v \quad (5a)$$

$$v = \frac{-2m\Omega s}{(ms^2 + cs + k - m\Omega^2)^2 + (2m\Omega s)^2} F_u + \frac{ms^2 + cs + k - m\Omega^2}{(ms^2 + cs + k - m\Omega^2)^2 + (2m\Omega s)^2} F_v \quad (5b)$$

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \mathbf{G}_d \begin{bmatrix} F_u \\ F_v \end{bmatrix} \quad (6)$$

Where \mathbf{G}_d is a 2×2 transfer function matrix.

$$\mathbf{G}_d = \frac{1}{k} \frac{1}{G_{dp}} \begin{bmatrix} G_{dz} & G_{dc} \\ -G_{dc} & G_{dz} \end{bmatrix} \quad (7)$$

With:

$$G_{dp} = \left(\frac{s^2}{\omega_0^2} + 2\xi \frac{s}{\omega_0} + 1 - \frac{\Omega^2}{\omega_0^2} \right)^2 + \left(2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \right)^2 \quad (8a)$$

$$G_{dz} = \frac{s^2}{\omega_0^2} + 2\xi \frac{s}{\omega_0} + 1 - \frac{\Omega^2}{\omega_0^2} \quad (8b)$$

$$G_{dc} = 2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \quad (8c)$$

- $\omega_0 = \sqrt{\frac{k}{m}}$: Natural frequency of the mass-spring system in rad/s
- ξ damping ratio

2.4 Constant Rotating Speed

To simplify, let's consider a constant rotating speed $\dot{\theta} = \Omega$ and thus $\ddot{\theta} = 0$.

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{1}{(ms^2 + (k - m\omega_0^2))^2 + (2m\omega_0 s)^2} \begin{bmatrix} ms^2 + (k - m\omega_0^2) & 2m\omega_0 s \\ -2m\omega_0 s & ms^2 + (k - m\omega_0^2) \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{\frac{1}{k}}{\left(\frac{s^2}{\omega_0^2} + \left(1 - \frac{\Omega^2}{\omega_0^2} \right) \right)^2 + \left(2 \frac{\Omega s}{\omega_0^2} \right)^2} \begin{bmatrix} \frac{s^2}{\omega_0^2} + 1 - \frac{\Omega^2}{\omega_0^2} & 2 \frac{\Omega s}{\omega_0^2} \\ -2 \frac{\Omega s}{\omega_0^2} & \frac{s^2}{\omega_0^2} + 1 - \frac{\Omega^2}{\omega_0^2} \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \quad (10)$$

When the rotation speed is null, the coupling terms are equal to zero and the diagonal terms corresponds to one degree of freedom mass spring system.

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \frac{\frac{1}{k}}{\frac{s^2}{\omega_0^2} + 1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \quad (11)$$

When the rotation speed is not null, the resonance frequency is duplicated into two pairs of complex conjugate poles. As the rotation speed increases, one of the two resonant frequency goes to lower frequencies as the other one goes to higher frequencies (Figure 2).

The magnitude of the coupling terms are increasing with the rotation speed.

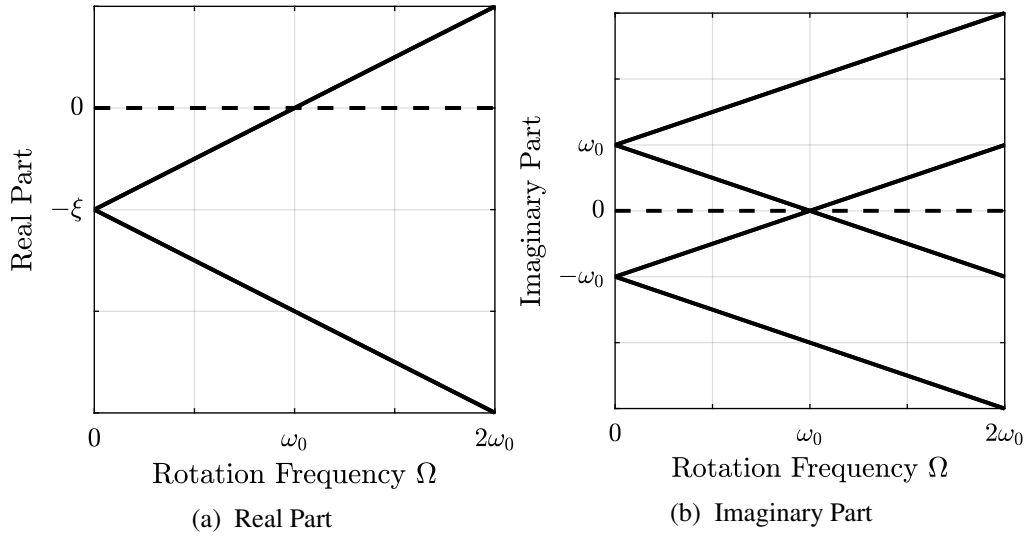


Figure 2: Campbell Diagram

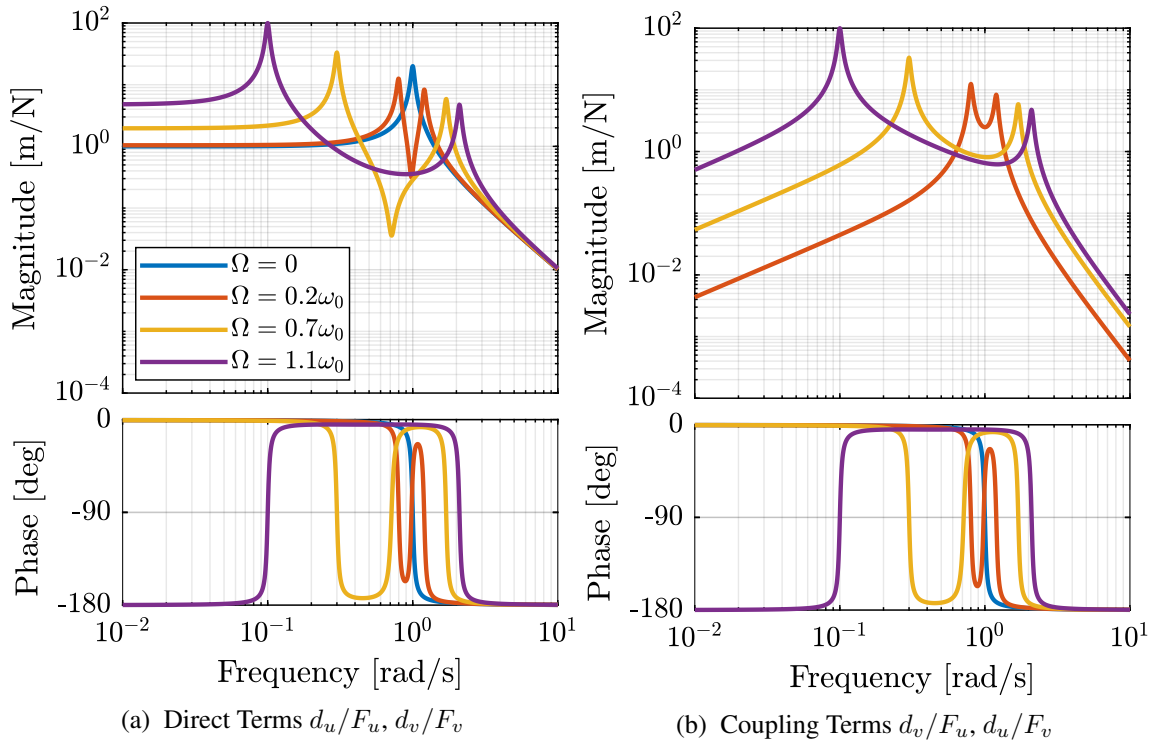


Figure 3: Dynamics

3 Integral Force Feedback

3.1 Control Schematic

Force Sensors are added in series with the actuators as shown in Figure 4.

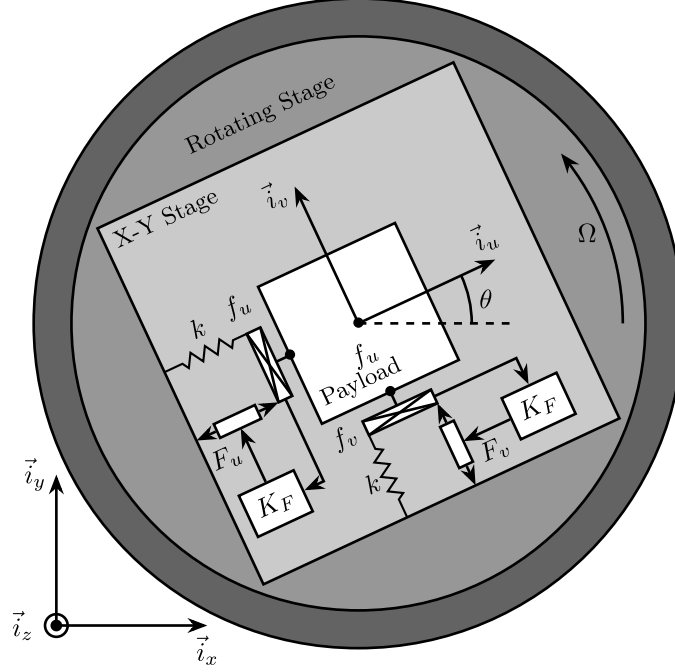


Figure 4: System with Force Sensors in Series with the Actuators. Decentralized Integral Force Feedback is used

3.2 Equations

The sensed forces are equal to:

$$\begin{bmatrix} f_u \\ f_v \end{bmatrix} = \begin{bmatrix} F_u \\ F_v \end{bmatrix} - (cs + k) \begin{bmatrix} d_u \\ d_v \end{bmatrix} \quad (12)$$

Which then gives:

$$\begin{bmatrix} f_u \\ f_v \end{bmatrix} = \mathbf{G}_f \begin{bmatrix} F_u \\ F_v \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} f_u \\ f_v \end{bmatrix} = \frac{1}{G_{fp}} \begin{bmatrix} G_{fz} & -G_{fc} \\ G_{fc} & G_{fz} \end{bmatrix} \begin{bmatrix} F_u \\ F_v \end{bmatrix} \quad (14)$$

$$G_{fp} = \left(\frac{s^2}{\omega_0^2} + 2\xi \frac{s}{\omega_0} + 1 - \frac{\Omega^2}{\omega_0^2} \right)^2 + \left(2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \right)^2 \quad (15)$$

$$G_{fz} = \left(\frac{s^2}{\omega_0^2} - \frac{\Omega^2}{\omega_0^2} \right) \left(\frac{s^2}{\omega_0^2} + 2\xi \frac{s}{\omega_0} + 1 - \frac{\Omega^2}{\omega_0^2} \right) + \left(2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \right)^2 \quad (16)$$

$$G_{fc} = \left(2\xi \frac{s}{\omega_0} + 1 \right) \left(2 \frac{\Omega}{\omega_0} \frac{s}{\omega_0} \right) \quad (17)$$

3.3 Plant Dynamics

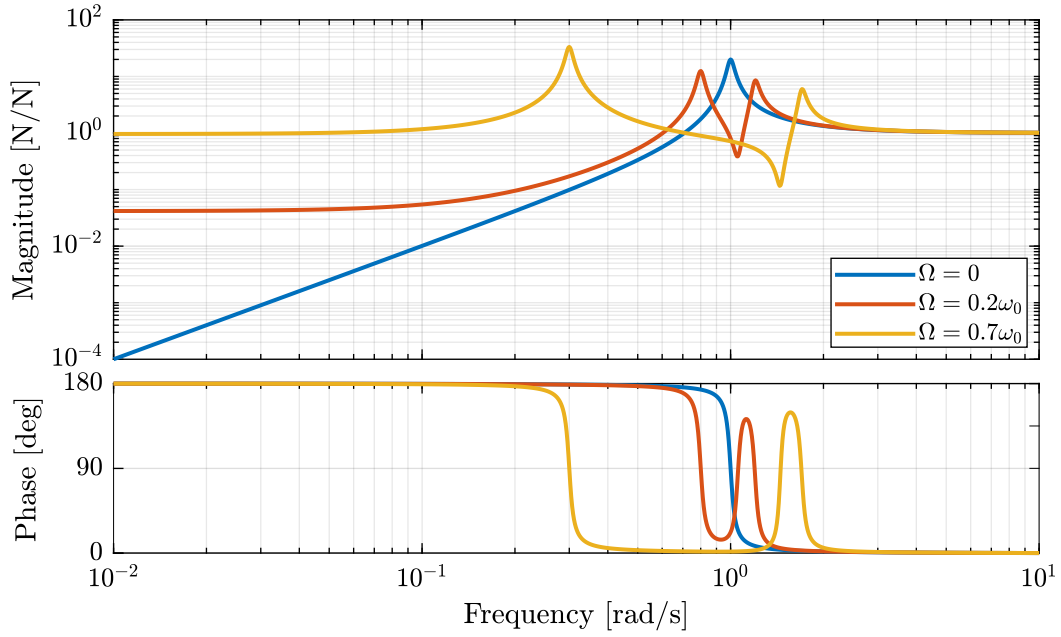


Figure 5: Figure caption

3.4 Integral Force Feedback

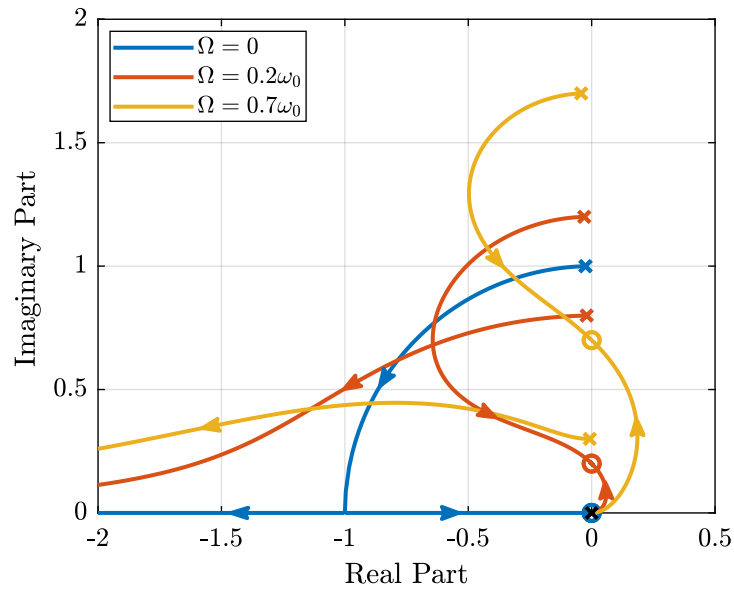


Figure 6: Root Locus

At low frequency, the gain is very large and thus no force is transmitted between the payload and the rotating stage. This means that at low frequency, the system is decoupled (the force sensor removed) and thus the system is unstable.

4 Integral Force Feedback with High Pass Filters

4.1 Modification of the Control Low

4.2 Feedback Analysis

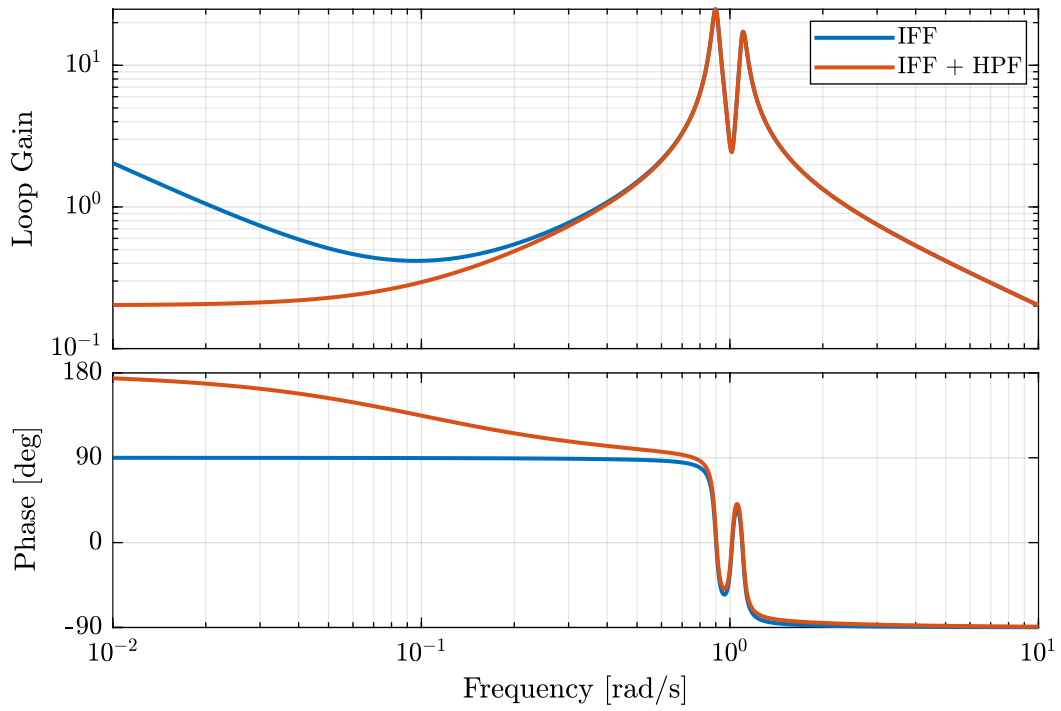


Figure 7: Figure caption

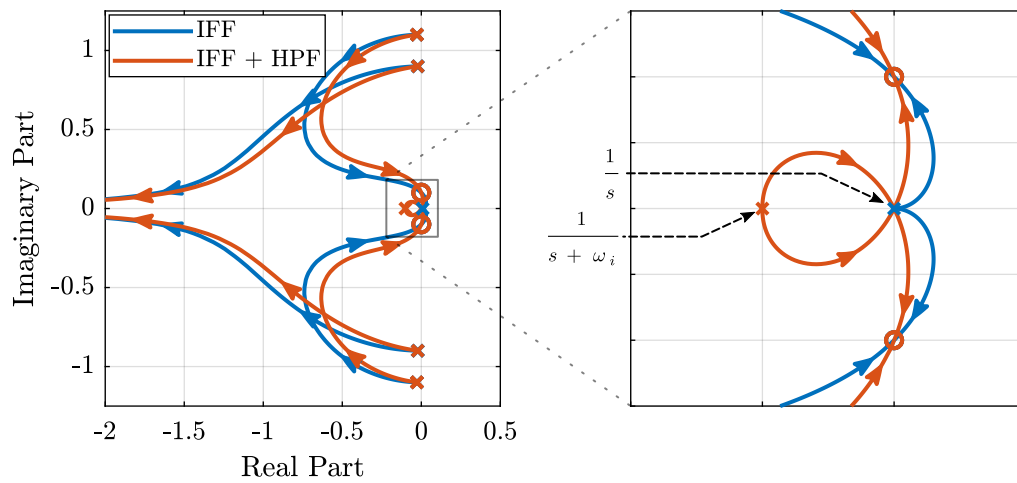


Figure 8: Figure caption

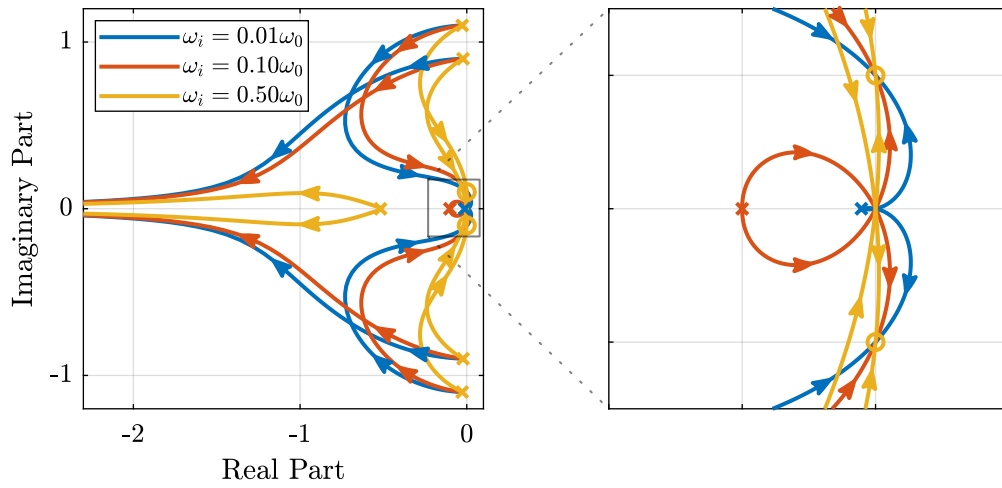


Figure 9: Figure caption

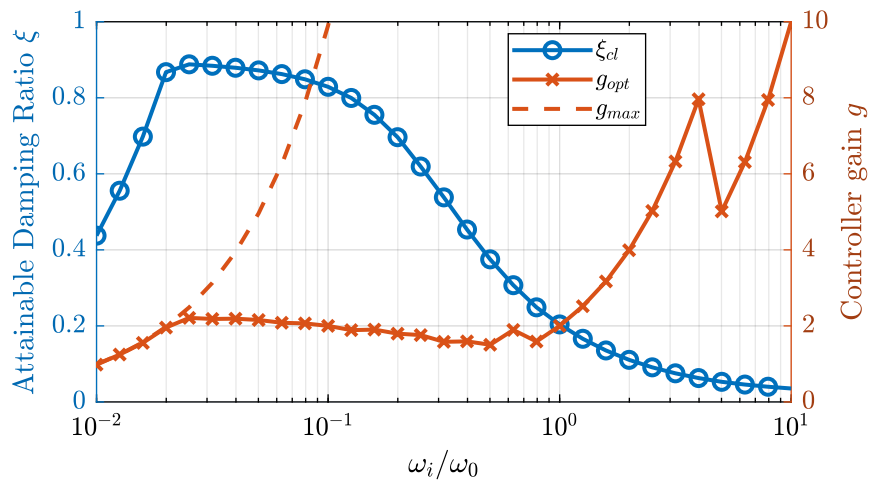


Figure 10: Figure caption

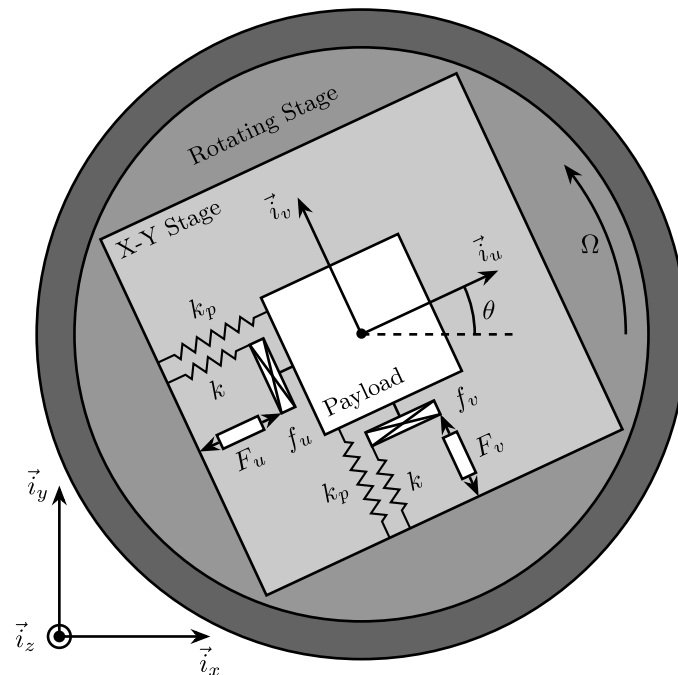


Figure 11: Figure caption

4.3 Optimal Cut-Off Frequency

5 Integral Force Feedback with Parallel Springs

5.1 Stiffness in Parallel with the Force Sensor

5.2 Effect of the Parallel Stiffness on the Plant Dynamics

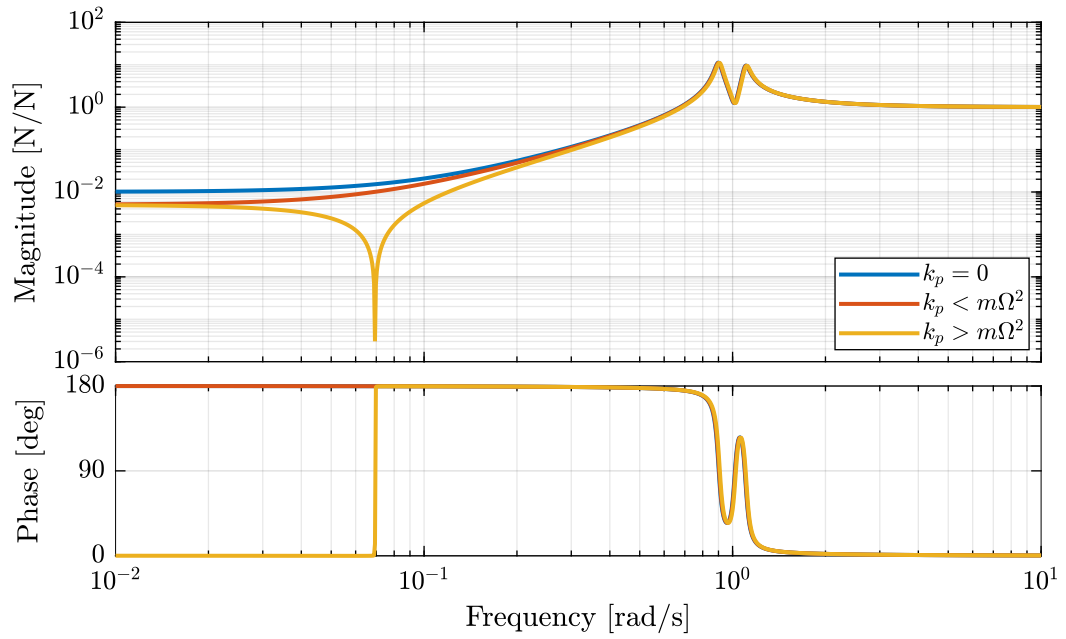


Figure 12: Figure caption

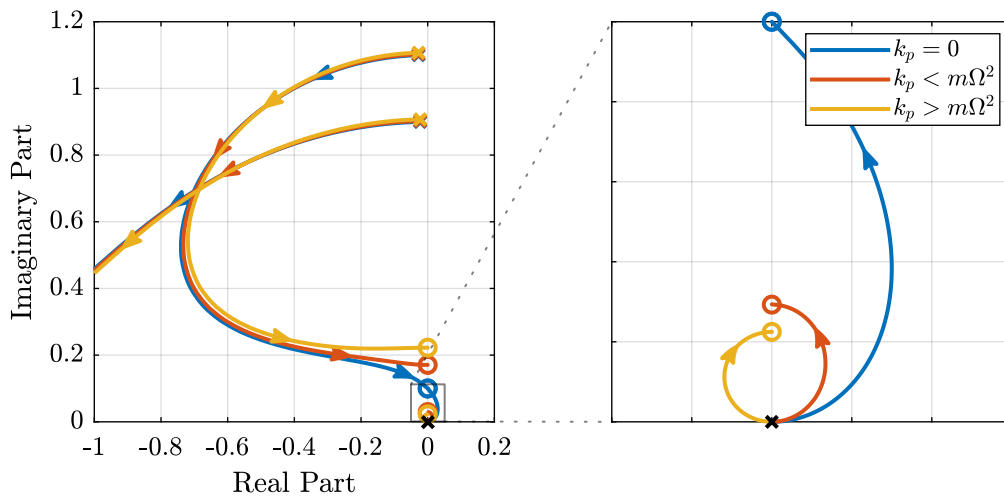


Figure 13: Figure caption

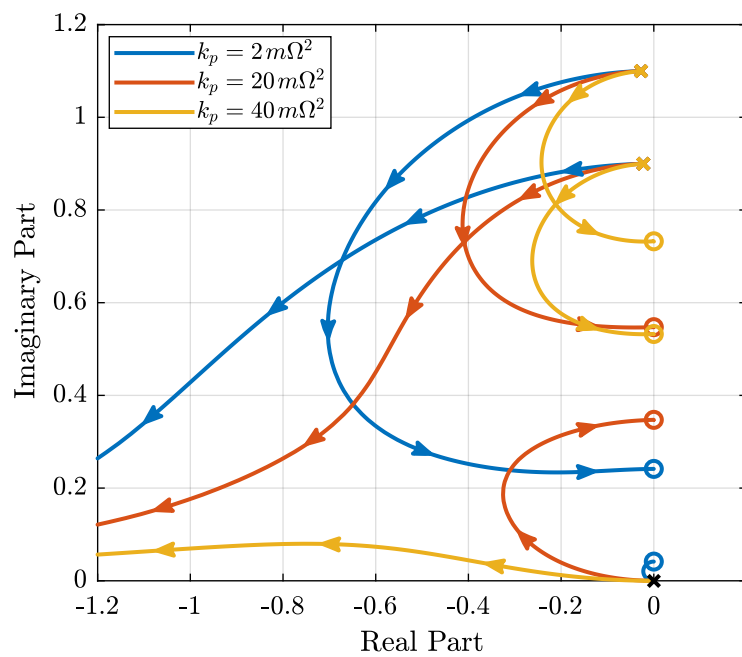


Figure 14: Figure caption

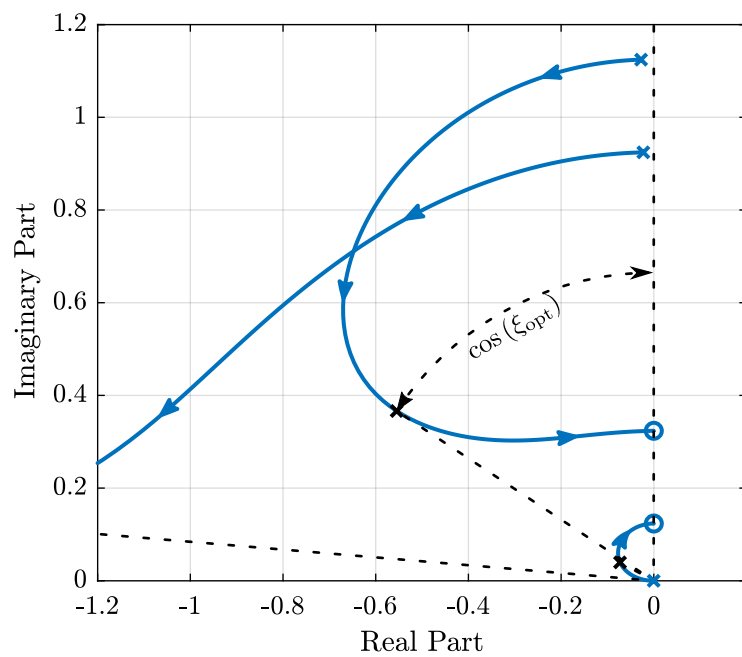


Figure 15: Figure caption

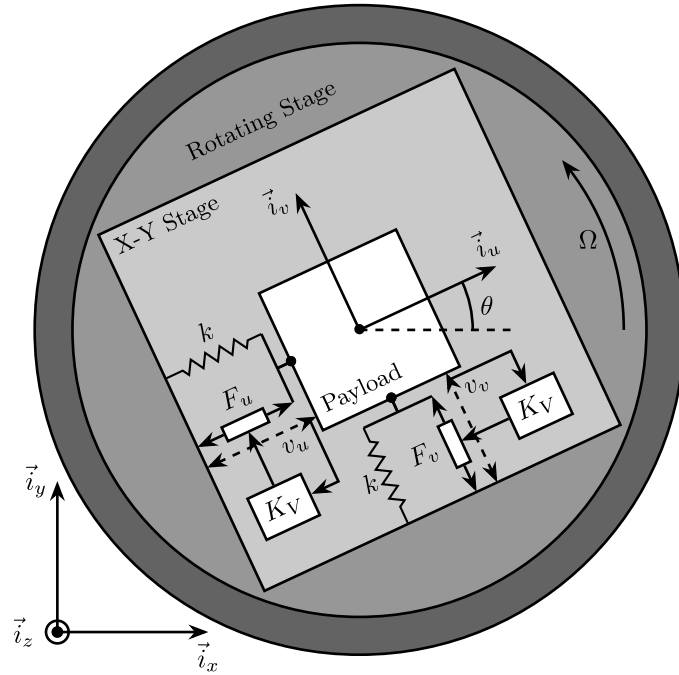


Figure 16: Figure caption

5.3 Optimal Parallel Stiffness

6 Direct Velocity Feedback

6.1 Control Schematic

6.2 Equations

6.3 Relative Direct Velocity Feedback

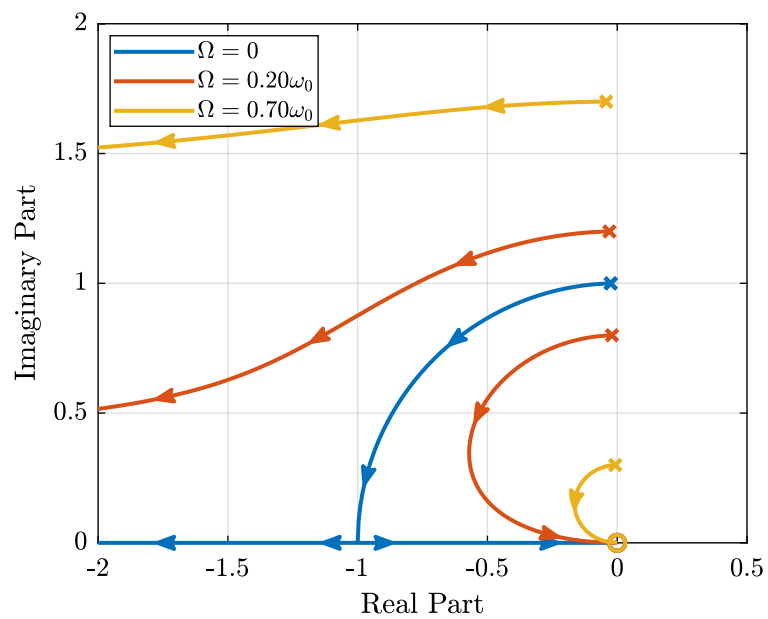


Figure 17: Figure caption

7 Comparison of the Proposed Active Damping Techniques for Rotating Positioning Stages

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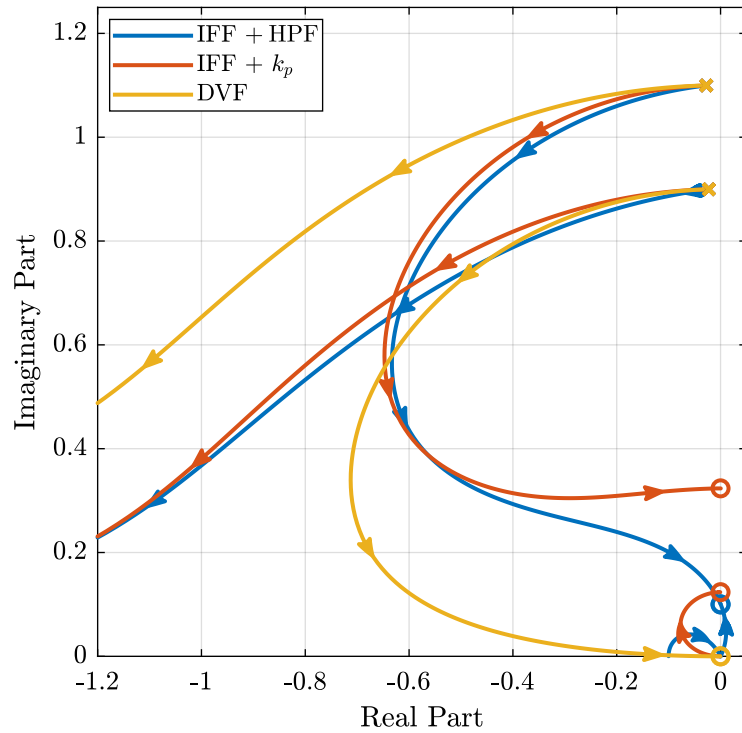


Figure 18: Figure caption

7.1 Transmissibility and Compliance

8 Conclusion

Acknowledgment

References

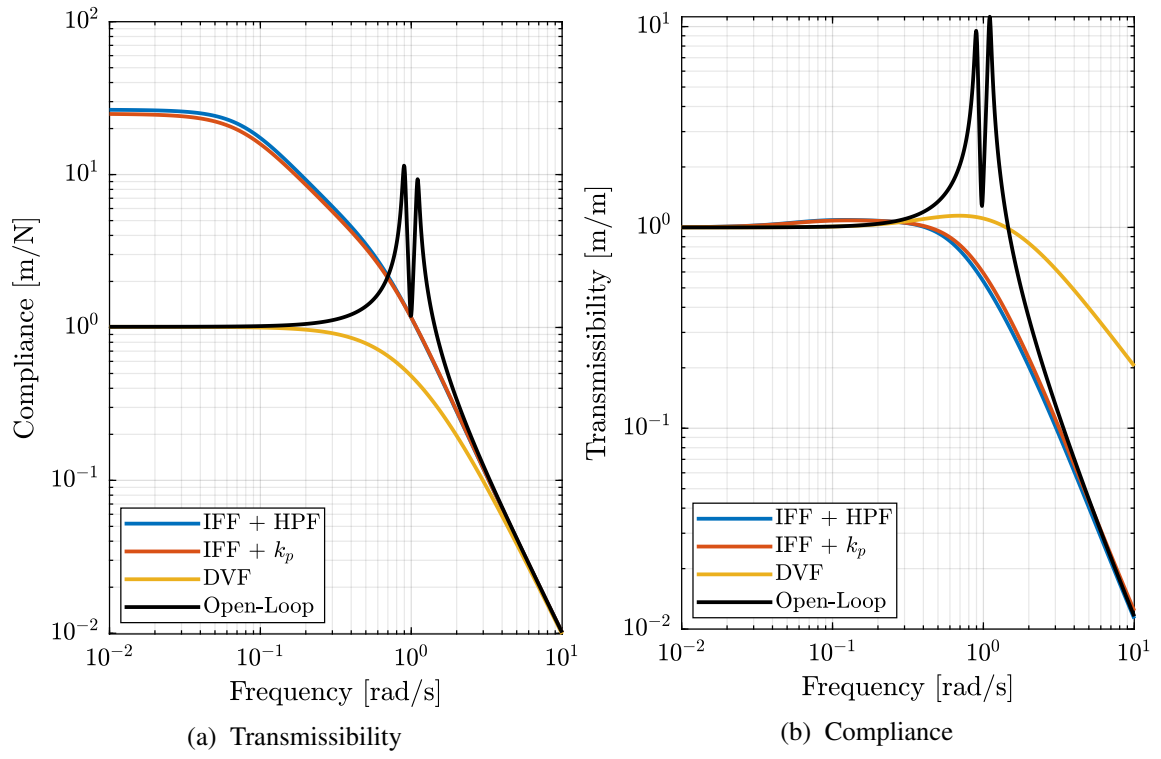


Figure 19: Comparison of the three proposed Active Damping Techniques