DCM - Dynamical Multi-Body Model

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Contents

1 System Identification

1.1 Kinematics (111 Crystal)

The reference frame is taken at the center of the 111 second crystal.

1.1.1 Interferometers - 111 Crystal

Three interferometers are pointed to the bottom surface of the 111 crystal.

The position of the measurement points are shown in Figure 1.1 as well as the origin where the motion of the crystal is computed.

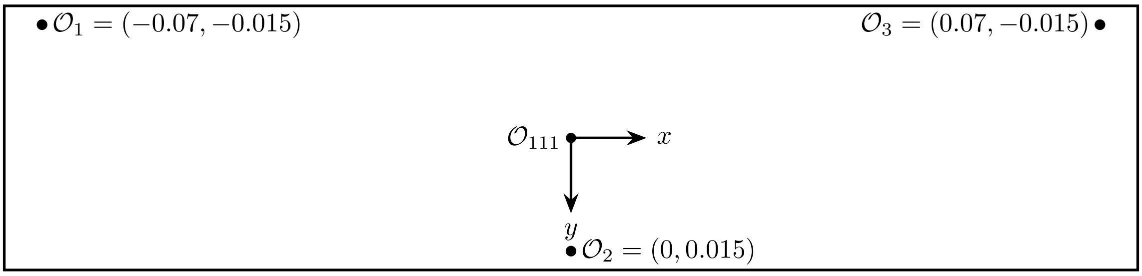

Figure 1.1: Bottom view of the second crystal 111. Position of the measurement points.

The inverse kinematics consisting of deriving the interferometer measurements from the motion of the crystal (see Figure 1.5):

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \boldsymbol{J}_{s,111} \begin{bmatrix} d_z \\ r_y \\ r_x \end{bmatrix} \tag{1.1}$$

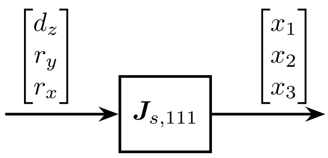

Figure 1.2: Inverse Kinematics - Interferometers

From the Figure 1.1, the inverse kinematics can be solved as follow (for small motion):

$$\boldsymbol{J}_{s,111} = \begin{bmatrix} 1 & -0.07 & -0.015 \\ 1 & 0 & 0.015 \\ 1 & 0.07 & -0.015 \end{bmatrix} \tag{1.2}$$

```
%% Sensor Jacobian matrix for 111 crystal
J_s_111 = [1, -0.07, -0.015
           1,  0,     0.015
           1,  0.07, -0.015];
```

Table 1.1: Sensor Jacobian $\boldsymbol{J}_{s,111}$

1.0	-0.07	-0.015
1.0	0.0	0.015
1.0	0.07	-0.015

The forward kinematics is solved by inverting the Jacobian matrix (see Figure 1.3).

$$\begin{bmatrix} d_z \\ r_y \\ r_x \end{bmatrix} = \boldsymbol{J}_{s,111}^{-1} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{1.3}$$

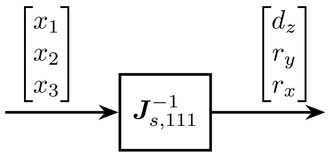

Figure 1.3: Forward Kinematics - Interferometers

Table 1.2: Inverse of the sensor Jacobian $\boldsymbol{J}_{s,111}^{-1}$

0.25	0.5	0.25
-7.14	0.0	7.14
-16.67	33.33	-16.67

1.1.2 Piezo - 111 Crystal

The location of the actuators with respect with the center of the 111 second crystal are shown in Figure 1.4.

Inverse Kinematics consist of deriving the axial (z) motion of the 3 actuators from the motion of the crystal's center.

$$\begin{bmatrix} d_{u_r} \\ d_{u_h} \\ d_d \end{bmatrix} = \boldsymbol{J}_{a,111} \begin{bmatrix} d_z \\ r_y \\ r_x \end{bmatrix} \tag{1.4}$$

Based on the geometry in Figure 1.4, we obtain:

$$\boldsymbol{J}_{a,111} = \begin{bmatrix} 1 & 0.14 & -0.1525 \\ 1 & 0.14 & 0.0675 \\ 1 & -0.14 & -0.0425 \end{bmatrix} \tag{1.5}$$

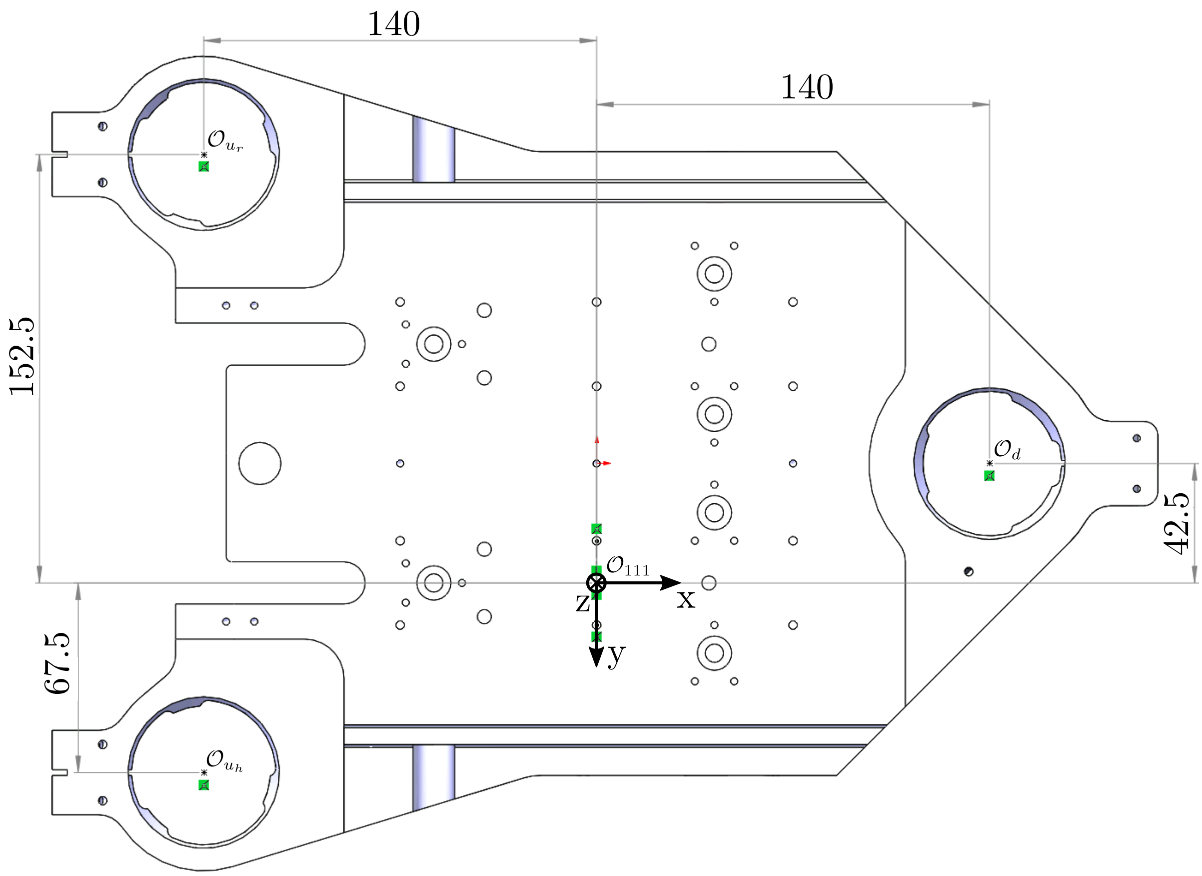

Figure 1.4: Location of actuators with respect to the center of the 111 second crystal (bottom view)

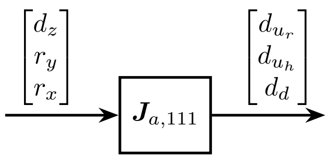

Figure 1.5: Inverse Kinematics - Actuators

Matlab
```matlab
%% Actuator Jacobian - 111 crystal
J_a_111 = [1,  0.14, -0.1525
           1,  0.14,  0.0675
           1, -0.14, -0.0425];
```

Table 1.3: Actuator Jacobian $\boldsymbol{J}_{a,111}$

1.0	0.14	-0.1525
1.0	0.14	0.0675
1.0	-0.14	-0.0425

The forward Kinematics is solved by inverting the Jacobian matrix:

$$\begin{bmatrix} d_z \\ r_y \\ r_x \end{bmatrix} = \boldsymbol{J}_{a,111}^{-1} \begin{bmatrix} d_{u_r} \\ d_{u_h} \\ d_d \end{bmatrix} \tag{1.6}$$

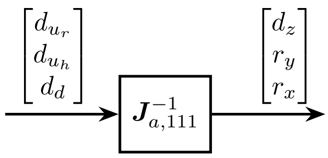

Figure 1.6: Forward Kinematics - Actuators for 111 crystal

Table 1.4: Inverse of the actuator Jacobian $\boldsymbol{J}_{a,111}^{-1}$

0.0568	0.4432	0.5
1.7857	1.7857	-3.5714
-4.5455	4.5455	0.0

1.2 Identification

Let's considered the system $\boldsymbol{G}(s)$ with:

- 3 inputs: force applied to the 3 fast jacks
- 3 outputs: measured displacement by the 3 interferometers pointing at the 111 second crystal

It is schematically shown in Figure 1.7.

The system is identified from the Simscape model.

Matlab
```matlab
%% Input/Output definition
clear io; io_i = 1;

%% Inputs
% Control Input {3x1} [N]
io(io_i) = linio([mdl, '/u'], 1, 'openinput');  io_i = io_i + 1;
% % Stepper Displacement {3x1} [m]
```

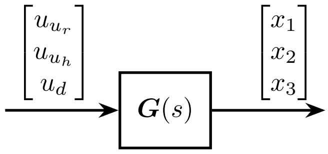

Figure 1.7: Dynamical system with inputs and outputs

```
% io(io_i) = linio([mdl, '/d'], 1, 'openinput');  io_i = io_i + 1;

%% Outputs
% Interferometers {3x1} [m]
io(io_i) = linio([mdl, '/int_111'], 1, 'openoutput'); io_i = io_i + 1;
```

Matlab
```
%% Extraction of the dynamics
G = linearize(mdl, io);
```

Matlab
```
size(G)
```

Results
```
size(G)
State-space model with 3 outputs, 3 inputs, and 24 states.
```

1.3 Plant in the frame of the fastjacks

Using the forward and inverse kinematics, we can computed the dynamics from piezo forces to axial motion of the 3 fastjacks (see Figure 1.8).

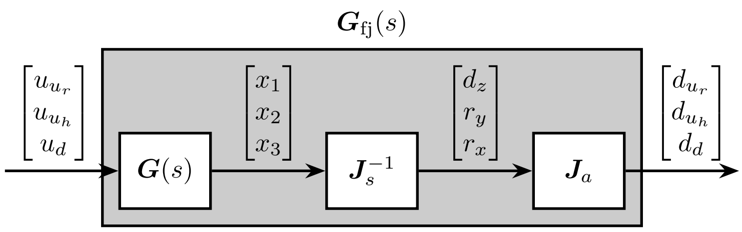

Figure 1.8: Use of Jacobian matrices to obtain the system in the frame of the fastjacks

Matlab
```
%% Compute the system in the frame of the fastjacks
G_pz = J_a_111*inv(J_s_111)*G;
```

The DC gain of the new system shows that the system is well decoupled at low frequency.

```
dcgain(G_pz)
```

Table 1.5: DC gain of the plant in the frame of the fast jacks $\boldsymbol{G}_{\text{fj}}$

4.4408e-09	2.6469e-12	9.6727e-13
2.6469e-12	4.4408e-09	9.6729e-13
9.5372e-13	9.5372e-13	4.4425e-09

The bode plot of $\boldsymbol{G}_{\text{fj}}(s)$ is shown in Figure 1.9.

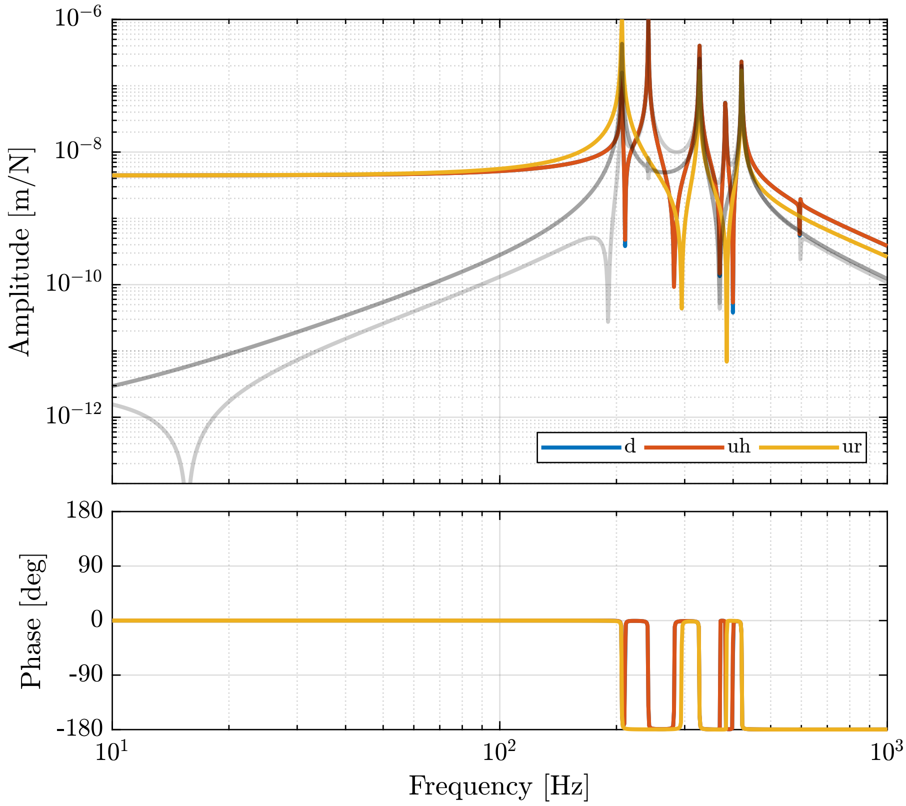

Figure 1.9: Bode plot of the diagonal and off-diagonal elements of the plant in the frame of the fast jacks

Important

Computing the system in the frame of the fastjack gives good decoupling at low frequency (until the first resonance of the system).